Banking, Liquidity and Bank Runs
in an Infinite Horizon Economy

Mark Gertler and Nobuhiro Kiyotaki
NYU and Princeton University
Recent financial crisis

Slow run on shadow banks from Summer 2007

Loss in subprime loans and related assets → Financial intermediaries loose capital → Spreads between liquid and illiquid assets expand

Fast run after Lehmann failure in September 2008

Securitized assets market freezes

Wholesale and retail funding contracts. Asset prices fall further

"Great Recession"
We develop a simple macro model of banking crisis

Financial accelerator / Credit cycles

Roll-over risk, or "Bank run"

Slow bank run \approx \text{An increase of the likelihood of run}

Macroeconomic conditions affect whether runs are feasible

Bank leverage ratio

Liquidation prices
Basic Model

Capital is either intermediated by banks or directly held by households

\[ K_t^b + K_t^h = \overline{K} \]

\[ \begin{align*}
\text{date t} & \quad \text{date t} + 1 \\
K_t^b \text{ capital} & \rightarrow \begin{cases} 
K_t^b \text{ capital} \\
Z_{t+1}K_t^b \text{ output}
\end{cases} \\
f(K_t^h) \text{ goods} & \rightarrow \begin{cases} 
K_t^h \text{ capital} \\
Z_{t+1}K_t^h \text{ output}
\end{cases}
\end{align*} \]

\[ f(K_t^h) = \frac{\alpha}{2} (K_t^h)^2 : \text{management cost } \alpha > 0 \]
\[ Q_t K^b_t = N_t + D_t \]
Deposit contract

Short term

Promised rate of return $\overline{R}_{t+1}$ is non-contingent

With run, the returns is the minimum of $\overline{R}_{t+1}$ and total realized bank assets per deposit

In Basic Model, bank run is unanticipated $\rightarrow$

Realized return: $R_{t+1} = \overline{R}_{t+1}$ : Promised return
Households maximize

\[ U_t = E_t \left( \sum_{i=0}^{\infty} \beta^i \ln C_{t+i}^h \right) \]

subject to:

\[ C_t^h + D_t + Q_t K_t^h + f(K_t^h) = Z_t W^h + R_t D_{t-1} + (Z_t + Q_t) K_{t-1}^h \]

\[ 1 = E_t (\Lambda_{t,t+1}) R_{t+1} \]

\[ 1 = E_t \left( \Lambda_{t,t+1} \frac{Z_{t+1} + Q_{t+1}}{Q_t + f'(K_t^h)} \right) \]

\[ \Lambda_{t,t+1} = \beta \frac{C_t}{C_{t+1}} \]
Many bankers

Each has an i.i.d. survival probability of $\sigma$

Banker consumes wealth upon exit: $c^b_t = n_t$

Preferences are linear in "terminal" consumption

$$V_t = E_t \left[ \sum_{i=1}^{\infty} \beta^i \sigma^{i-1} (1 - \sigma) c^b_{t+i} \right]$$

Each exiting banker replaced by a new banker with an endowment $w^b = n_t$

Net worth $n_t$ of surviving bankers

$$n_t = (Z_t + Q_t) k^b_{t-1} - R_t d_{t-1}$$
$Z_t$ is realized

<table>
<thead>
<tr>
<th>B/S of Bank</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Asset: $Q_t k_t^b$</td>
<td>Deposit: $d_t$</td>
</tr>
<tr>
<td>Net worth: $n_t$</td>
<td></td>
</tr>
</tbody>
</table>

Incentive constraint:

$$\theta Q_t k_t^b \leq V_t$$

Figure 1: Timing

Date $t$

Continue: $V_t$

Divert $\theta Q_t k_t^b$

Bankrupt

Repay $R_{t+1} d_t$

Retain $n_{t+1}$

Exit or continue

Date $t+1$
Bank chooses $k_t^b$ and $d_t$ to maximize

$$V_t = \beta E_t[(1 - \sigma)n_{t+1} + \sigma V_{t+1}]$$

Bank chooses "leverage multiple" $\phi_t = \frac{Q_t k_t^b}{n_t}$ to maximize

$$\frac{V_t}{n_t} = \psi_t = \beta E_t \left\{ (1 - \sigma + \sigma \psi_{t+1}) \frac{n_{t+1}}{n_t} \right\}$$

$$= \beta E_t \left\{ (1 - \sigma + \sigma \psi_{t+1}) \left[ \phi_t \left( \frac{Q_{t+1} + Z_{t+1}}{Q_t} - R_{t+1} \right) + R_{t+1} \right] \right\}$$

$$= \mu_t \phi_t + \nu_t$$

subject to $\theta \phi_t \leq \psi_t$. →

$$\phi_t = \frac{\nu_t}{\theta - \mu_t}, \text{ if } \mu_t \in (0, \theta)$$
Aggregate leverage constraint

\[ Q_t K_t^b = \phi_t N_t \]

Aggregate net worth

\[ N_t = \sigma \left[ (Z_t + Q_t) K_{t-1}^b - R_t D_{t-1} \right] + (1 - \sigma) w^b \]

Goods market

\[ C_t^h + (1 - \sigma) \left[ (Z_t + Q_t) K_{t-1}^b - R_t D_{t-1} \right] + f(K_t^h) = Z_t \overline{K} + Z_t W^h + (1 - \sigma) w^b \]
Bank Runs

Ex ante, zero probability of a run

If depositors do not roll over their deposits ("run"), the bank sells its capital to households who are less efficient in managing capital.

In addition to an equilibrium without run, bank run equilibrium exists if:

\[(Z_t + Q_t^*) K_{t-1}^b < R_tD_{t-1}\]

\(Q_t^* \equiv \text{the liquidation price of the bank’s assets}\)
After a bank run at $t$:

$$K_t^h = \bar{K},$$

$$N_{t+1} = (1 - \sigma)w^b + \sigma(1 - \sigma)w^b$$

$$N_s = \sigma \left[ (Z_s + Q_s)K_{s-1}^b - R_sD_{s-1} \right] + (1-\sigma)w^b, \ \forall \ s \geq t+2$$

Household condition for direct capital holding $\rightarrow$

$$Q_t^* = E_t \left\{ \sum_{i=1}^{\infty} \Lambda_{t,t+i}[Z_{t+i} - f'(K_{t+i}^h)] \right\} - f'(\bar{K})$$
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount rate</td>
<td>0.99</td>
</tr>
<tr>
<td>Bankers survival prob.</td>
<td>0.95</td>
</tr>
<tr>
<td>Seizure rate</td>
<td>0.19</td>
</tr>
<tr>
<td>Household managerial</td>
<td>0.008</td>
</tr>
<tr>
<td>Serial correlation</td>
<td>0.95</td>
</tr>
<tr>
<td>Steady state productivity</td>
<td>0.0126</td>
</tr>
<tr>
<td>Bankers endowment</td>
<td>0.0011</td>
</tr>
<tr>
<td>Household endowment</td>
<td>0.045</td>
</tr>
</tbody>
</table>
Table 2: Steady State Values

<table>
<thead>
<tr>
<th>Steady State Values</th>
<th>Baseline</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>1</td>
</tr>
<tr>
<td>Q</td>
<td>1</td>
</tr>
<tr>
<td>$c_h$</td>
<td>0.055</td>
</tr>
<tr>
<td>$c_b$</td>
<td>0.0036</td>
</tr>
<tr>
<td>$K^h$</td>
<td>0.31</td>
</tr>
<tr>
<td>$K^b$</td>
<td>0.69</td>
</tr>
<tr>
<td>$\phi$</td>
<td>10</td>
</tr>
<tr>
<td>$R^k$</td>
<td>1.0504</td>
</tr>
<tr>
<td>$R^h$</td>
<td>1.0404</td>
</tr>
<tr>
<td>$R$</td>
<td>1.0404</td>
</tr>
</tbody>
</table>
FIGURE 3: A Recession in the Baseline Model; No Bank Run Case
Figure 4: Ex-Post Bank Run in the Baseline Model

- $y$: %Δ from ss
- $kb$: %Δ from ss
- $Q$: %Δ from ss
- RUN: %Δ from ss
- $Q^*$: %Δ from ss
- $\phi^*$: %Δ from ss
- ch: %Δ from ss
- cb: %Δ from ss
- ER$_b^*$-R: Ann. Δ from ss

Legend:
- Red: No Run Recession
- Blue: unanticipated run
Extension: Anticipated Bank Runs

Deposit returns $R_{t+1} = \begin{cases} \overline{R}_{t+1} & \text{if no bank run} \\ x_{t+1} \overline{R}_{t+1} & \text{if bank run} \end{cases}$

$x_{t+1} = \text{Min} \left[ 1, \frac{(Q_{t+1}^* + Z_{t+1}) K_t^b}{\overline{R}_{t+1} D_t} \right]$}

Household attaches the probability of bank run as

$p_t = 1 - E_t(x_{t+1})$

FONC for deposits is

$1 = \overline{R}_{t+1} [(1 - p_t) E_t (\Lambda_{t,t+1}) + p_t E_t (\Lambda_{t,t+1}^* x_{t+1})]$
Bank’s leverage \( \phi_t = \frac{Q_t k_t^b}{n_t} \) maximizes

\[
\frac{V_t}{n_t} = \psi_t = \\
\beta(1-p_t)E_t \left\{ (1-\sigma+\sigma\psi_{t+1}) \left[ \phi_t \left( \frac{Q_{t+1} + Z_{t+1}}{Q_t} - \overline{R}_{t+1} \right) + \overline{R}_{t+1} \right] \right\}
\]

subject to \( \theta \phi_t \leq \psi_t \).

An increase in likelihood of run is contractionary in two ways

leverage \( \phi_t \) declines when the franchise value falls

\( N_{t+1} \) decreases even without run since \( \overline{R}_{t+1} \) increases
Figure 5: Recession with positive probability of a run

- $p$, $y$, $kb$
- $Q$, $\phi$, $n$
- $ER^b - R^d$, $R^d - R^{free}$, $R^{free}$

Legend:
- Blue: Recession with positive probability of run
- Red dashed: No Run Recession
Figure 6: Recession with positive Run Probability and Ex-Post Run
Description: The data series for Credit spreads is the Excess Bond Premium as computed by Gilchrist and Zakrasjek (2012); Bank Equity is the S&P500 Financial Index. The model counterparts are the paths of $E(R_b - R_d)$ and $V$ as depicted in Figure 6 normalized so that their steady-state values match the actual values in 2007 Q2.
Some Remarks About Policy

Deposit insurance makes depositors careless → Bank will divert the assets

Capital requirement reduces bank risk-taking and likelihood of bank run

Can increase intermediation cost if capital is costly to raise

Lender-of-last resort stabilizes liquidation price

May reduce the likelihood of run

But increase the leverage multiple ex ante and the financial accelerator