SQUARING THE CIRCLE:
CLEARING ARRANGEMENTS IN OVER-THE-COUNTER
DERIVATIVES MARKETS

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February 2010

Abstract

Central Counterparty (CCP) clearing appears to be the silver bullet in the debate on structural reform of Over-the-Counter (OTC) derivatives markets. Curtailing systemic risk through a revision of the existing market structure and risk sharing mechanisms is the ultimate objective of regulators and policy makers. Resistance by market players is broadly considered as collective action and strategic behavior issue of the private sector which could be mitigated by concerted government action.

However, the argument is made by market players and scholars alike that existing decentralized risk sharing mechanisms – if deployed stringently and comprehensively – could offer lower costs and higher benefits than the propagated centralized alternative. Our paper develops a framework for a comparative analysis of alternative multilateral clearing arrangements for OTC derivatives markets.

The paper adds to the academic research on clearing arrangements by explicitly modeling the prevailing tiered market structure within which the dealers in their dual capacity as liquidity providers and prime brokers form the inner circle of the market. We square the circle by mapping the market structure to a uniform matrix representation. Analyzing various clearing mechanisms like close-out netting, multilateral prime broker give-ups and tear-ups, and finally CCP clearing arrangements allows us to compare their impact on netting efficiency as a main driver for expected average collateral cost and replacement cost risk.

The analytical framework developed in this paper is supposed to serve as a sound basis for numerical simulations of structural dynamics and distributional effects. However, its thorough construction already delivers some initial findings and insights we share in this paper. The findings suggest that with respect to netting efficiency multilateral arrangements could deliver results comparable to the central clearing through a CCP.

Keywords: derivatives market organization, clearing arrangements, default risk sharing, netting efficiency, prime brokerage, market regulation

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1 INTRODUCTION

Central Counterparty (CCP) clearing appears to be the silver bullet in the debate on structural reform of Over-the-Counter (OTC) derivatives markets. Curtailing systemic risk through a revision of the existing market structure and risk sharing mechanisms is the ultimate objective of regulators and policy makers. Resistance by market players is broadly considered as collective action and strategic behaviour issue of the private sector which could be mitigated by concerted government action.

However, the argument is made by market players and scholars alike that existing decentralized risk sharing mechanisms – if deployed stringently and comprehensively – could offer lower costs and higher benefits than the propagated centralized alternative. Our paper develops a framework for a comparative analysis of alternative multilateral clearing arrangements for OTC derivatives markets.

The paper adds to the academic research on alternative clearing arrangements by explicitly modelling the prevailing tiered market structure within which the dealers in their dual capacity as liquidity providers and prime brokers form the inner circle of the market. We square the circle by mapping the market structure to a uniform matrix representation. Analyzing various clearing mechanisms like close-out netting, multilateral prime broker give-ups and tear-ups, and finally CCP clearing arrangements allows us to compare their impact on netting efficiency as a main driver for expected average collateral cost and replacement cost risk.

The findings presented in this paper suggest that with respect to netting efficiency multilateral arrangements could deliver results comparable to the central clearing through a CCP. We will challenge these initial findings in future research by replenishing our framework with a comprehensive numerical simulation of the distributional effects and structural dynamics to fully assess the implications of alternative clearing arrangements. The framework developed in this paper seems to be a sound basis.

The structure of the paper is as follows. The second chapter focuses on industrial organisation of OTC derivatives markets and the perceived transformational changes. Moreover, the rationale and status of European and US reform plans are discussed. The chapter concludes with a review of some academic contributions to the debate. Chapter three develops the analytical framework for our comparative analysis. We consecutively apply and analyse the
impact of different clearing arrangements. Chapter four concludes with a summary of our findings. It also presents current limitations of our framework and an outlook on the continuation of our project.

2 ORGANIZATIONAL STRUCTURES AND TRANSFORMATIONAL CHANGES IN OTC DERIVATIVES MARKETS

2.1 Structural Features Emerging from Trading Relations

The on-exchange (order book) and OTC derivatives markets architectures differ in terms of type of order flow interactions, market access and required intermediation. In the OTC derivatives market the order flow interactions are mostly of bilateral nature as transactions are privately negotiated and typically executed over telephone. Market access is segmented – end-users typically do not trade directly with each other, but utilize intermediaries.

The eminent feature of OTC derivatives markets continues to be the dominant role of sell side dealers\(^1\) in their dual capacity as prime brokers (i.e. counterparty risk takers and leverage providers) and market makers (i.e. product structurers and liquidity providers). Operationally and with regard to cross asset class risk management, only the globally diversified large dealers with large scale prime brokerage services seem to be able to offer the economies of scale and scope to attract institutional customers and flows from the buy side. Consequently, there is a fairly high degree of concentration of trading liquidity and counterparty risk with a relatively small group of dealers. In principle, this holds across all OTC derivatives markets.

Trading between sell side dealers in the inter-dealer market constitutes the largest part of OTC trading; in the dealer-to-customer markets the same large dealers trade with buy side customers and other less capitalised banks. Moreover, as the dealers take over the role of prime brokers, back-to-back trades between a prime broker, hedge fund client and executing dealer are generated. All described trading dependencies lead to a market structure characterised by a high share of intermediated deals and high concentration of market activity within the inner circle of the market formed by a small group of dealers. According to OCC’s quarterly reporting in OCC (2009), top five dealers concentrate ca. 90% trades in the US OTC

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\(^1\) A dealer is a firm that enters into transactions as a counterparty on both sides of the market. It is recognised that OTC derivatives dealers are primarily large international banks. The appellation sell side refers to a dealer’s elemental function of “selling liquidity” to the so called buy side. Sell side encompasses financial institutions offering trading services to institutional investors like asset managers, hedge funds etc. but also to commercial hedgers and corporate clients referred to as buy side. Refer to Harris (2003) for more detailed profile of buy and sell side market players.
derivatives market. Weekly data from the DTCC Trade Information Warehouse for Credit Default Swaps provided at DTCC (2010) confirms the statistics with ca. 80% of the market by value occurring between dealers, ca. 20% between the dealers and buy side customers and only 0.1% inter-customers.

2.2 Existing Post-trade Arrangements in OTC Derivatives Markets

Today’s OTC derivatives markets deploy the following bilateral and multilateral post-trade arrangements: close-out netting, tear-up cycles, prime broker give-ups and CCP clearing. In this subchapter we will discuss the market penetration of these different arrangements. Prime brokerage will be covered in the following subchapter.

Bilateral clearing is based on position netting solely between direct trading counterparties. The most important feature of the bilateral relationship, as fixed in the Master Agreements ruling the largest part of the OTC derivatives market, is close-out netting. Close-out netting is an arrangement to settle all contracted but not yet due liabilities to and claims on an institution by one single payment immediately upon the occurrence of one of a list of defined events, such as for example the appointment of a liquidator to that institution.²

In the current market practice, the so called tear-up and portfolio compression services play a role similar to traditional ring clearing arrangement as they allow for a multilateral trade termination. In an offsetting ring, the participants’ positions are multilaterally netted preserving individual counterparty exposure. Accordingly, tear-up cycles result in terminating the offsetting trades submitted by participants. The offsetting algorithm generates a set of bilateral contracts between participants that provides the same net exposures but lower gross exposures. The reduced gross credit exposure on the market participants’ books mitigates both counterparty credit risk and operational burden.

Under CCP clearing netting by novation takes place as the original obligations of the trading parties are discharged and replaced with the new obligations against a CCP. In doing so,

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² Refer to Bergman et al. (2004) for a definition and further discussion of close-out netting. See also BIS (2007) for legal implications. Although netting is regarded as a risk mitigation tool it carries with it some legal risk. This is because netting may not be legally enforceable in all jurisdictions in which a financial institution operates and each type of netting has varying degrees of enforceability in the event of default.
trades with the CCP are netted on a multilateral basis. Moreover, a loss sharing mechanism caters for a loss mutualisation between members of the CCP in case of one’s default.³

Accelerated through the recent credit crisis, developments in the market for Credit Default Swaps (CDS) already provides empirical evidence that the purely bilateral clearing has been increasingly substituted by the multilateral netting mechanism in a form of portfolio compression and multilateral tear-ups. According to the latest figures from the Depository Trust & Clearing Corp in DTCC (2010), tear-ups have contributed to a reduction of the notional size of the credit derivatives market to about $25.3 trillion, from estimates of $62 trillion in 2009.⁴ Figure 1 illustrates the developments in notional outstanding and terminations through tear-up cycles which was most pronounced in 2008.

![Graph showing notional amount outstanding and CDS tear-ups](image)

**Figure 1:** Notional amount outstanding of the Credit Default Swap market and the terminated CDS trades (trillion USD).


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³ A CCP is a risk sharing mechanism and like all risk sharing mechanisms incurs costs arising from information asymmetries (adverse selection and moral hazard). We resign from modeling the costs leaving it for a future research.

⁴ The relative size of the torn up interest rates derivative market positions was, however, lower. (TriOptima, 2010)
Strongly demanded by regulators and politicians, the accelerated proliferation of CCPs for OTC derivatives has started as well after the recent credit crisis reaching 20% of the CDS and 60% of the interest rate derivatives markets volumes.\(^5\)

In summary, Figure 2 below gives an impression of the overall joint impact of the different incumbent netting, clearing and collateralization arrangements in OTC derivatives markets. Notional amount outstanding provides a gross measure of a market size, whereas the cost of replacing existing contracts is rather expressed in the gross market value. The dramatic reduction from ca. 605 trillion USD notional to 25.4 trillion USD would be achieved with application of multilateral termination services. Gross credit exposure represents positive market value after taking into account bilateral close-out netting agreements i.e. it measures netted credit exposure. App. 66% of credit exposure is collateralised, which results in remaining unsecured exposure of ca. 1.3 trillion USD.

![Figure 2: OTC derivatives market size as of June 2009 according to different measures.](source: BIS (2009) and ISDA Margin Survey (2009))

2.3 Central Role of Prime Brokerage

The expansion of the prime brokerage into OTC derivatives markets has changed the structure of the relationships between the buy and sell side. With the soaring activity of hedge funds, prime brokerage, as noticed in a the Bank of International Settlement report, has become

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\(^5\) Both, CDS and interest rate derivatives markets make more than 70% of the total OTC derivatives market volume. (BIS, 2009)
increasingly popular in the OTC derivatives market. BIS (2007) describes the prime brokerage function as follows: “In a prime brokerage arrangement, a prime broker agrees to intermediate specified eligible transactions between a hedge fund client and any of a list of approved executing dealers”. The kind of intermediation aims at taking market access advantage of the prime broker by a hedge fund client, which encompasses access to multi-dealer pricing and liquidity with post-trade processing centralised by a prime broker.

![Figure 3: Prime brokerage agreement](image)

Figure 3 gives an illustration of a typical multilateral prime broker give-up arrangement between a hedge fund on the buy side, a sell side prime broker and a sell side executing dealer. Once an executing dealer and the buy side customer have agreed to a trade, both must each notify the prime broker of the terms of the deal. If the prime broker accepts the trade it becomes counterparty to the two back-to-back trades, one with the hedge fund and one with the executing dealer. In consequence, the prime broker becomes exposed to the credit risk of the opposing parties – executing dealer and hedge fund – whereas the exposure to market risk remains with original counterparties to the trade.

The expansion of the prime brokerage into OTC derivatives market has impacted the market structure in diverse ways. Firstly, the number of trades on gross basis doubles, as multilateral prime brokerage arrangements generate back-to-back trades. From the market structure perspective the prime brokerage leads to a higher dealer-client market concentration. Secondly, the prime broker concentrates risk. Although its position resulting from a trade is neutral as the contracts offset, the counterparty risk to both the hedge fund and executing dealer remains.

Similar to the role of a CCP, the trades with multiple executing dealers are assigned to a prime broker, who thus takes over the counterparty credit risk vis-à-vis the hedge funds’ counterparties, Also like a CCP, prime brokers centralise information about clients' accounts and carries the responsibilities of counterparty credit quality monitoring. Therefore, prime brokers must ensure that they have adequate information and control to protect against counterparty credit risk arising both from the client and from the executing dealer. It is worth
to notice that, in opposite to a CCP, the prime brokerage agreement assumes no default loss sharing with the other parties subject to the multilateral arrangement. A prime broker guarantees the contract performance up to its equity capital.

2.4 Regulatory Agenda for OTC Derivatives Market Reform

The regulatory agenda for OTC derivatives market reform focuses on four main risk reporting and mitigation tools: increased usage of CCP clearing, buy side access to CCP facilities, wider use of compression services and trade reporting by means of trade information repositories.

The CCP clearing, in which a Central Counterparty stands as the seller to every buyer and the buyer to every seller, has been identified by regulators and lawmakers as one way to reduce systemic risk in the OTC market. In the US, the Senate is reviewing a derivatives bill that would mandate the central clearing of the standardised OTC instruments. The European Commission has publicly supported similar reforms, but UK regulator the Financial Services Authority announced that while it supported greater standardisation of credit derivatives and more counterparty risk managed, it did not see the need for CCP clearing.

The buy side access to CCPs for OTC derivatives has been targeted as a further aim on the regulatory agenda. Jackson and Manning (2007) recognise that buy side access to CCP clearing platforms enable to lower the systemic risk as it decreases the loss concentration in case of market participant’s default. Therefore, risk-averse policy makers support the customer access to central clearing platforms. In opposite, as the wider CCP membership base leads to higher costs for the dealers, the largest market participants stayed, for a long time, reluctant to opening the CCP to the buy side customers. However, under regulatory demands, Swapclear and ICE Trust US clearing houses have allowed the indirect customer participation. Customer asset segregation and portability of customer positions in case of dealers’ default have been addressed by regulators as key requirements in this context.

The increased usage of compression services has been recognised by the policy makers as to further enhance resilience of the OTC derivatives market. Portfolio compression allows collapse superfluous positions, thus reducing the associated counterparty risk and cutting the market’s aggregate notional amount of OTC trades. Prior to the recent crisis, active market participants typically held large simultaneous long and short positions referencing the same underlying. These redundant positions posed significant unnecessary counterparty exposure and offered no material economic benefit.
For non clearing eligible OTC derivatives the regulators have envisaged mandatory reporting to trade repositories. Whereas for CDSs a trade information warehouse has already managed to attract more than 90% of the market (DTCC, 2010), the market participants have recently committed to develop comprehensive trade repositories for other types of OTC derivatives.

2.5 Review of Recent Academic Contributions to the Debate

The subject of clearing has not received much attention from academic researchers. This has changed in the aftermath of the recent financial crisis. However, only a few academic contributions have aimed at an end-to-end quantitative assessment of alternative post trade processing arrangements for OTC derivatives markets.

Jackson and Manning (2007) compare, in a simulation approach, the implication of the shift between different clearing arrangements. Under the assumption of agent homogeneity, the netting efficiency achieved under a multilateral ring agreement equals that under a CCP clearing and generally increases with the number of participants but at decreasing rate. Tiered CCP membership structures is according to Jackson and Manning a natural response on the different credit quality of the CCP members and involves cost of margin rate tailoring. If a margin rate applied by CCP corresponds to an average default probability of the member groups, the more creditworthy participants bear the higher cost than lower credit quality agents.

The model of Jackson and Manning has, however, its limitations hindering the direct transfer of the results on the OTC derivatives market. First, the initial setup for netting efficiency comparison under the diverse clearing arrangements is based on a two-dimensional matrix representation only. Such eminent features of the OTC market like segmented market access and prime brokerage stay beyond the scope of their analysis. Secondly, they do not – at least not explicitly – model a market with different asset classes nor does their model accommodate the coexistence and combination of different clearing arrangements.

Duffie & Zhu (2009) analyse in a simple model whether adding a central counterparty for a particular asset class improves the efficiency of counterparty risk mitigation and collateral demands, relative to bilateral netting between pairs of dealers. The results show that for plausible cases where the number of dealers under a CCP arrangement is relatively small, adding a CCP for one asset class like CDS for example may actually reduce netting efficiency and thereby lead to an increase in collateral demand and average exposure to default. However, the model does not consider the implications of a tiered market structures nor does it take into account any other multilateral arrangements other than CCP clearing.
The slow adoption of CCP clearing for OTC derivatives – at least prior to the recent credit crisis – is explained in Pirrong (2006) as a result of a higher importance of private information in the OTC than in the exchange traded markets. Central counterparties and large OTC derivatives dealers represent different types of intermediation and two competitive ways of taking advantage of economies of scale in default risk. Information asymmetry observed in the OTC market makes it more costly to share default risks through a clearing house than allocate risk through the large dealers. The information asymmetry results from the dealers’ insight into the client’s activity in various banking areas, which gives them additional information on the client’s vulnerability to default. Moreover, dealers that intermediate OTC transactions possess valuation know-how regarding innovative financial products, which enables them to develop valuation models at a lower cost than third parties like e.g. clearing houses.

Pirrong’s argumentation may explain the dealer resistance to adopt CCPs for OTC derivatives observed until the recent credit crisis. With the credit crisis there is regulatory pressure to implement CCP clearing in the OTC derivatives markets. Also as a consequence of the credit crisis, banks’ traditionally high credit ratings and strong balance sheets deteriorated and their superior skill to assess creditworthiness of the counterparties (and by definition better risk pricing) has been put in question.

In the context of the recent public debate on systemic risk and the call for CCP clearing facilities, Pirrong added to its earlier work by further examining information asymmetries. In Pirrong (2009) he points out various redistributive effects resulting from CCP clearing in OTC derivatives markets like for example the implications of a CCP guaranteeing the positions of a clearing member’s customer. Thus, a CCP effectively insure non-members against default risk. He also includes the introduces the notion that bilateral counterparties have better information than a CCP regarding counterparty balance sheet risk and can price counterparty risks to reflect risk differentials, whereas a CCP treats all members alike.

To our knowledge, the available research has neither provided an end-to-end modelling of the constitutive features of the OTC derivatives market organization nor a comprehensive comparative analysis of structural dynamics and distribution effects of different clearing arrangements. The work presented in this paper and its envisaged continuation utilizing numerical simulation techniques aims at closing this gap in the research on default risk mitigation mechanisms.
3 ANALYTICAL FRAMEWORK

3.1 Approach

This section develops the analytical framework for our research on the impact of different clearing arrangements in OTC derivatives markets. Along with the engineering of the framework we consecutively apply and analyse the impact of the underlying mechanics of the following types of bilateral and multilateral clearing arrangements: close-out netting, multilateral prime broker give-ups, multilateral tear-ups and, finally, central CCP clearing with and without buy side participation. The framework is supposed to be the foundation for future research using comprehensive numerical simulations to analyse structural dynamics, default and liquidity risk dispersion in the market and resulting systemic risk issues. However, its thorough construction already delivers some initial findings and insights we share in this paper. Before applying and comparing the alternative arrangements we define an intuitive initial metrics for comparison and lay out the formal representation of the market structure.

3.2 Initial Metrics for Comparison

For all alternative clearing arrangements analysed hereafter, the metrics of interest for comparison is the change in magnitude of expected replacement cost borne by surviving agents and the change in magnitude of the cost to protect against replacement cost risk. Replacement cost risk arises during the period between the opening of a contract position and the closing of the position through final settlement. It reflects the cost to agents when being forced to replacing contract positions on which a counterparty defaults.6

Agents can mitigate replacement cost risk by collecting margin collateral in the form of cash or securities from their counterparties during the pre-settlement period. Providing margin collateral implies opportunity cost for the agents involved. Since we assume risk-neutrality, all agents’ calculus to choose a particular clearing arrangement should be based on the expected total of both the replacement cost and the opportunity cost for margin collateral to protect against replacement cost risk.

Our focus on the comparative analysis of ideal type clearing arrangements necessitates some simplification of the assumed margining regimes between agents – and of course between agents and the CCP clearing introduced at a later stage. Without loss of generality we hence assume a perfect margining regime with the following properties:

6 For a definition and further discussion of replacement cost risk refer to BIS (2007).
(1) **Variation Margin:** There is a real-time mark-to-market regime for all positions $X_{i,j,l}$ between any agent $i$ and any agent $j$ in some asset class $l$ with immediate and frictionless exchange of the respective variation margin credits and debits in the form of cash collateral. Consequently, we apply a factor of $m^V = 1$ for the variation margin calculation across all agents and clearing arrangements. Since we always assume a complete system of mirrored positions between any agent $i$ and any agent $j$ with $X_{i,j,l} = -X_{j,i,l}$ we can also assume, that the net amount of the circulating variation margin collateral in the system is always 0. Consequently, we can for now abstract away from any funding and opportunity cost for variation margin collateral.\(^7\)

(2) **Initial Margin:** There is a fully symmetrical and complete Initial Margin regime for all bilateral net positions with positive and negative market values.\(^8\) Both, agent $i$ and agent $j$, hold mirrored positions with $X_{i,j,l} = -X_{j,i,l}$ and will alternately exchange the same amount of initial margin collateral to cover for potential future changes in the market value of the respective positions. Let $m^I > 0$ be the factor for initial margin. Then, agent $i$ will be required to provide $|X_{i,j,l}| \cdot m^I$ in additional margin collateral to agent $j$ and vice versa. Since we assume homogeneity of agents with respect to default risk and identical and independent distributions for changes in market values of asset classes, we for now apply a uniform factor $m^I > 0$ across agents and across asset classes.

\(^7\) Indeed, by abstracting away from any opportunity cost for variation margin collateral we for now also ignore any margin pooling effects possibly provided by multilateral clearing arrangements. In particular, a CCP as a central venue allows agents to economize on collateral pooling and pooling of settlement liquidity on the cash as well as on the securities side.

\(^8\) In practice, many OTC derivatives master agreements still provide for the posting of initial margin in only one direction. Sometimes there is no immediate initial margin required but rather an unsecured threshold. A further common practice in OTC markets is the adjustment of initial margin requirements to credit quality. For example, the collateral up-tick on a one notch rating downgrade is estimated at $1 \text{ billion}$ for Morgan Stanley’s derivatives business. The AIG bailout case exemplifies the dramatic implication such practice can have. The S&P downgrade of AIG by three notches in September 2008 triggered an immediate call for additional $20 \text{ billion}$ in margin collateral by AIG’s counterparties. Finally, the US treasury bailed out AIG by supporting the immediate funding of $32 \text{ billion}$ in collateral calls.

Most CCP have a very rigid but single-sided margining regime within which they to not post initial margins to members. For some early groundwork on alternative approaches to CCP margining of different product categories refer to Kupiec and White (1996). Refer to Gibson and Murawski (2008) for an analysis of the impact of collateralization and margining of derivatives on a banks’ welfare, default risk and trading volume.
Under the assumption that there is no difference between the funding cost for and the returns earned on margin collateral, we apply a uniform opportunity cost rate of \( \gamma \). The opportunity cost for agent \( i \) of providing collateral on a net position \( X_{i,j} \) is defined as:

\[
\gamma_i = |X_{i,j}| \cdot \gamma \cdot \gamma
\]

(1)

Against the background of the assumptions made above on the margining regimes, we can now define the expected replacement cost in a bilateral setting with some agent \( i \), a defaulting agent \( j \) and a position in some asset class \( l \). A replacement cost loss for the surviving agent \( i \) will only occur if there is a coincidence of the following events:

1. A default by agent \( j \) to perform on her variation margin obligations from the position with agent \( i \) other than for technical reasons.
2. An adverse change in the position’s market value of \( v^l \) caused by a corresponding change of the underlying risk variable in excess of the per-unit value \( m' \) of additional margin collateral posted by the defaulting agent \( j \) to agent \( i \).
3. Both the sign of the market value of the position held \( X_{i,j} \) and the sign of the change in market value \( v^l \) are the same.

From the assumptions (1) to (3) above follows that in case of agent \( j \) defaulting a replacement cost loss arises for agent \( i \) if and only if the following inequality holds:

\[
X_{i,j} \cdot v^l > |X_{j,i}| \cdot m'
\]

(2)

We generalize to define the expected replacement cost by drawing both the change in market values in some asset class \( v^l \) and the market value of the bilateral net contract position \( X_{i,j} \) from normal distributions:

\[
v^l \sim N(\mu_{v^l}, \sigma_{v^l}^2) \text{ with } \mu_{v^l} = 0 ; \sigma_{v^l}^2 \neq 0
\]

(3)

\[
X_{i,j} \sim N(\mu_X, \sigma_X^2) \text{ with } \mu_X = 0 ; \sigma_X^2 \neq 0
\]

(4)

We further assume that both are independent with \( \delta_{v^l} = \delta_{X} = 0 \). Hence, we can define the expected replacement cost \( \rho \) for agent \( i \) in a bilateral setting as the difference between the expected adverse change in market value of the position and the expected value of additional margin collateral posted times the default probability \( e_j \) of agent \( j \):
\[ \rho_{i,j} = \varepsilon_j \cdot \mathbb{E}( \max \{X_{i,j} \cdot V_i - |X_{i,j}| \cdot m^\Delta, 0 \} ) \]  

(5)

Taking the collateral cost \( \gamma_{i,j} \) on expected additional margin and the expected replacement cost \( \rho_{i,j} \) derived in the bilateral setting, we can generalize our metrics for comparison and formulate the following corollary under the assumption, that except for the market values of the net position exposures all variables are independent of the particular clearing arrangements.

**Corollary 1:** Both, the average expected replacement cost \( \rho \) to be borne by surviving agents after netting and collateral and the average additional margin collateral cost \( \gamma \), will be positively correlated to the average, across agents and asset classes, expected absolute market value of contract positions denoted by \( \varphi \):

\[ \rho := \rho(\varphi, v^A, m^a, ...) \quad \text{with} \quad \frac{\Delta \rho}{\Delta \varphi} < 0 \]  

(6)

\[ \gamma := \gamma(\varphi, m^a, c^m, ...) \quad \text{with} \quad \frac{\Delta \gamma}{\Delta \varphi} < 0 \]  

(7)

The impact of alternative clearing arrangements will, ceteris paribus, depend on a change in \( \varphi \). Hence, our comparative analysis will initially focus on reductions in \( \varphi \) caused by the application of different clearing arrangements.

### 3.3 Initial Matrix Representation

The framework for the comparative analysis of alternative clearing arrangements is set by a matrix representation. By using a matrix representation we follow the earlier approaches taken in Jackson and Manning (2007) and Duffie and Zhu (2009). However, our approach departs in explicitly transforming the initial matrix to represent structural changes caused by alternative clearing arrangements.

In the context of our initial analysis there is no need to explicitly model agents with differing default probabilities. However, the tiered structure of markets is – among other reasons – also a means to deal with varying credit quality of agents. Sell side dealers and prime brokers lever their balance sheet to offer risk intermediation for agents with lower credit quality – not just on the buy side. Similarly, restricted access to CCP clearing by means of tiered clearing structures, where more risky non-clearing members are not eligible for direct membership but rather have to access the CCP through more creditworthy clearing members, is a common structural feature of CCP in exchange-traded derivatives markets.

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matrix represents the market value of the bilateral net number of outstanding contracts awaiting settlement between agent \(i\) and agent \(j\) in some asset class \(l\). In the following we refer to it as net position exposure \(X_{i,j,l}\). This initial matrix will be the baseline reference for the comparative analysis. It is defined as follows:

\[
X_{\text{nonosc}} := \left( X_{i,j,l} \right)_{i=1,\ldots,n; j=1,\ldots,n; l=1,\ldots,k} \tag{8}
\]

As all \(X_{i,j,l}\) are net bilateral positions, the matrix \(X_{\text{nonosc}}\) is negatively symmetric about the diagonal with \(X_{i,j,l} = -X_{j,i,l}\) for all \(i \neq j\) and \(X_{i,i,l} = 0\) for all \(i = j\).

We now decompose \(X_{\text{nonosc}}\) to comprise four partial matrices to account for a market structure with \(s\) sell side agents and \(b\) buy side agents, i.e. we redefine the matrix as follows:

\[
X_{\text{nonosc}} = \begin{pmatrix}
X_{\text{sss}} & X_{\text{sdb}} \\
X_{\text{bsd}} & X_{\text{bbs}}
\end{pmatrix}_{n=s+b} \tag{9}
\]

The symmetry property implies that the partial buy-side-to-sell-side matrix results form a transformation of the respective sell-side-to-buy-side partial matrix, i.e. \(X_{\text{bsd}} = (-1)X_{\text{sdb}}^T\). Representing the observed market conduct of not allowing for direct buy-side-to-buy-side relations the partial buy-side-to-buy-side matrix is a zero matrix, i.e. \(X_{\text{bbs}} = 0\).

The symmetry property of \(X_{\text{nonosc}}\) and the blanking of \(X_{\text{bbs}}\) allows a reduction in the dimension of the problem. With \(s\) sell side dealers, \(b\) buy side clients, and \(k\) asset classes, there are \(k \times s \times (s-1)/2\) market values distributions to be specified in \(X_{\text{sss}}\) and \(k \times s \times b\) exposure distributions to be specified in either \(X_{\text{sdb}}\) or \(X_{\text{bsd}}\) to determine all bilateral net positions exposures.

All \(X_{i,j,l}\) to populate the respective partial matrices continue to be independent and drawn from a normal distribution. In taking the absolute market values of all net position exposures \(|X_{i,j,l}|\) and drawing correspondingly on a truncated normal distribution above the squeezed interval \([0,\ldots,\infty)\) we can calculate the average, across sell side agents, expected absolute position exposure of some sell side agent:
\[
\varphi_{ss}^a = \frac{1}{s} \left( \sum_{i=1}^{s} \sum_{j \neq i}^{s} \sum_{l=1}^{k} E\left( |X_{i,j,l}| \right) \right) = (s-1) \cdot k \cdot \frac{2}{\sqrt{\pi}} \sigma_x
\]  

(10)

The labelling with \( \alpha \) indicates that the term refers to the baseline matrix before applying any clearing arrangements. Applying the same calculus to the buy-side-to-sell-side and sell-side-to-buy-side partial matrices delivers:

\[
\varphi_{ssb}^a = \frac{1}{s} \left( \sum_{i=1}^{s} \sum_{j=i+1}^{s} \sum_{l=1}^{k} E\left( |X_{i,j,l}| \right) \right) = b \cdot k \cdot \frac{2}{\sqrt{\pi}} \sigma_x
\]  

(11)

\[
\varphi_{bbs}^a = \frac{1}{b} \left( \sum_{i=1}^{s} \sum_{j=i+1}^{s} \sum_{l=1}^{k} E\left( |X_{i,j,l}| \right) \right) = s \cdot k \cdot \frac{2}{\sqrt{\pi}} \sigma_x
\]  

(12)

3.4 Close-out Netting Agreements

Almost all trades and the resulting positions of sell and buy side market participants in the OTC derivatives markets are governed by bilateral master agreements. Close-out netting provisions are a pivotal element of such agreements. Close-out netting provides special legal protection towards insolvent counterparties for all open contract positions held across derivatives asset classes with a counterparty.\(^{11}\) To demonstrate the impact of such bilateral netting agreements let us assume that some agent \( i \) holds a set of contract positions with positive market values \( X_{i,j,l}^+ \) and negative market values \( X_{i,j,l}^- \) in some asset class \( l \) with an agent referenced with \( j \).

Firstly, close-out netting allows agents to net opposite contract positions in a particular asset class so that the bilateral position exposure between \( i \) and \( j \) in some asset class \( l \) can be reduced to the market value of the net contract position. Secondly, close-out netting allows agents to net off positions across all derivatives asset classes. Mutatis mutandis, the bilateral positions with positive and negative market values can also be reduced to a single net position so that the final net position between \( i \) and \( j \) across \( k \) asset classes is

\[
X_{i,j} = \sum_{l=1}^{k} \left( X_{i,j,l}^+ - X_{i,j,l}^- \right).
\]

---

\(^{11}\) Refer to Bergman et al. (2005) for a comprehensive discussion of the legal status and the economic implications of close-out netting provisions for derivatives and other off-balance sheet contracts.
As regards our metrics for comparison we now can calculate the average expected absolute market values for sell side and buy side agents after applying close-out netting on all position exposures across asset classes denoted by $\beta$. The calculations for the different partial matrices deliver:

$$\varphi_{\text{ss}}^\beta = \frac{1}{s} \left( \sum_{i=1}^{s} \sum_{j \in j \in k} E \left( \sum_{l=1}^{k} X_{i,j,l} \right) \right) = (s-1) \cdot \sqrt{k} \cdot \sqrt{\frac{2}{\pi}} \cdot \sigma_X$$ (15)

$$\varphi_{\text{sb}}^\beta = \frac{1}{s} \left( \sum_{i=1}^{s} \sum_{j = s+1}^{s+b} E \left( \sum_{l=1}^{k} X_{i,j,l} \right) \right) = b \cdot \sqrt{k} \cdot \sqrt{\frac{2}{\pi}} \cdot \sigma_X$$ (16)

$$\varphi_{\text{bs}}^\beta = \frac{1}{b} \left( \sum_{i=s+1}^{s+b} \sum_{j=1}^{s} E \left( \sum_{l=1}^{k} X_{i,j,l} \right) \right) = s \cdot \sqrt{k} \cdot \sqrt{\frac{2}{\pi}} \cdot \sigma_X$$ (17)

We can now derive changes in $\varphi$ caused by the application of close-out netting in defining the netting ratio $\tau$ as the quotient of the respective average expected position exposures. This delivers for the three relevant partial matrices:

$$\tau_{\text{ss}}^\beta = \frac{\varphi_{\text{ss}}^\beta}{\varphi_{\text{ss}}^\alpha} = \frac{\varphi_{\text{sb}}^\beta}{\varphi_{\text{sb}}^\alpha} = \frac{\varphi_{\text{bs}}^\beta}{\varphi_{\text{bs}}^\alpha} = \frac{1}{\sqrt{k}}$$ (24)

Obviously, the netting ratio for the corresponding buy side related partial matrices is identical to the ratio of the sell-side-to-sell-side matrix. Based on this, we can formulate the following corollary on the application of close-out netting arrangements:

**Corollary 2:** If applied radically across all agents in all market segments the netting efficiency after close-out netting increases in the number of asset classes considered for close-out netting arrangements but with decreasing marginal increments.

In addition to this finding on netting efficiency, close-out netting reveals both structural dynamics and distributional implication. It changes the distribution of default losses by giving derivatives counterparties a priority claim vis-à-vis other creditors in case of bankruptcy.\(^\text{12}\)

---

\(^\text{12}\) There is a fund of research work on the legal and economic implications of close-out netting provisions for derivatives and other off-balance sheet contracts. In summary, there is a controversial discussion on the overall impact. Refer for example to Bergman et al. (2005), Bliss and Kaufmann (2005), Edwards and Morrison (2005) for a comprehensive discussion of the legal status and the economic implications of close-out netting provisions.
But more importantly, it unfolds structural dynamics by setting strong incentives to consolidate derivatives business with an ever smaller number of sell side counterparties.\(^{13}\)

The first-order consequence of economizing on close-out netting and the related collateral pooling and operational efficiencies is the strong incentive to consolidate OTC derivatives business with a single large sell side dealer. This creates a high level of customer retention and economies of scale and scope draw more and more business of buy side agents to the chosen sell side dealer. This, in turn, potentially increases the concentration of replacement cost risk in the market. As a second-order consequence, sell side dealers in their capacity as liquidity provider benefit on the trading side from drawing more and more liquidity to their trading operations. This pooling of liquidity, in turn, attracts more and more counterparties and, consequently, reinforces the first-order implication.\(^{14}\) Along the arguments in Bliss and Kaufmann (2006) we can at this point draw the preliminary conclusion by saying, that it is not clear whether close-out netting comes without a cost. It leads to a reduction in the replacement cost risk in the market. However, it also provokes a highly concentrated risk profile in the market and, hence, increases systemic risk. Consequently, our future research will focus on the structural dynamics and allocation of replacement cost and default risks in the market.

### 3.5 Multilateral Prime Broker Give-up Arrangements

Prime brokerage is a business branch of sell side agents offering a bundled package of services to buy side customers. A prime broker’s service portfolio primarily targets hedge fund clients and may range from custody, securities financing and lending, cash management, derivatives clearing to reporting and sometimes to capital introduction.\(^{15}\) The prime brokers’ major source of income is the spread they earn on financing the customer’s long and short cash, security, and derivative positions. In some cases, they charge explicit fees for clearing...

---

\(^{13}\) According to the regular reporting in OCC (2009), for example, the five largest US commercial banks represent 97% of the total banking industry notional amounts and 88% of industry net current credit exposure in OTC derivatives. Refer to Duffie (2009) for a discussion of concentration issues and failure mechanisms of large dealer firms.

\(^{14}\) Pirrong (2006) provides a comprehensive discussion of the exploitation of scale economies and asymmetric information by large dealer firms. He concludes that private information and netting arrangements create scale and scope economies that could make it economical to eschew the sharing of default risk in a CCP – even for standardized and liquidly traded OTC derivatives. See also subchapter 2.5 above.

\(^{15}\) The unique nature of hedge funds makes management of counterparty credit risk exposures to them intrinsically more difficult, both for the regulated prime brokerage institutions and finally for policymakers concerned with systemic risk. Cases like LTCM provided empirical evidence for the issue. Refer for example to Kamphu et al. (2007) for a detailed analysis of the issue.
and/or other services. Prime brokers also generate income by hypothecating the client’s long securities portfolios and pledged collateral pools.\footnote{Re-hypothecation practices used to be a major source of income for prime brokers. The issues with non-segregated collateral accounts surfacing in the aftermath of the Lehman bankruptcy caused a dramatic change. A run for segregated accounts by prime brokerage customers followed and led to a massive deleveraging. For example, the total collateral received by the largest four prime brokers that was allowed to be pledged onward (hypothecation) shrunk by $1.7 trillion globally in 2008 and remains at levels as low as 30-40\% of the pre-Lehman times. See also Singh and Aitken (2009).}

As to our comparative analysis we focus on the prime brokers’ derivatives clearing function. Here, the business advantage to a buy side agent of using a prime broker is that the prime broker provides a centralized clearing facility, and the buy side agent’s exposures and collateral requirements are netted across all deals handled by the chosen prime broker. On the contrary, the disadvantage used to result from the fact that prime brokers are always principal counterparts in the deals cleared with their buy side customers and, hence, if a buy side client wants to open or close positions the customer would be constraint to deal with the originally chosen sell side prime broker. Consequently, access to market liquidity used to be constraint to the chosen prime broker conduit and buy side agents used to be tied to particular sell side counterparties capitalizing on their dual capacity as liquidity providing dealer and clearing prime broker.

With prime broker give-up arrangements are an already prevalent multilateral mechanism to dilute this concentration imperative on the sell side. With give-up arrangements within which buy side agents are allowed to trade with some sell side agents other than their prime broker and still having all trades given up to their chosen prime broker, there is a commonly used mechanism in the market allowing for the segregation of clearing and trading relations. In our framework we will deploy such multilateral give-up arrangements radically, i.e. all sell side and buy side agents participate in multilateral give-up arrangements. Hence, we can stick to the implicit assumption on competitive market structures on the clearing and trading level. This is, in turn, the prerequisite for our underlying assumption on the normal distribution of the original net contract positions.

For simplicity and without loss of generality we further assume that all sell side agents act in a dual capacity as prime broker and executing dealer and, hence, buy side agents can execute trades with any sell side agent whereupon such trades will be given-up to the chosen prime broker. Finally, we assume that buy side agents maximize the netting effect by selecting only one sell side agent as prime broker. This assumption ignores a reasonable calculus of buy side agents to diversify counterparty risk by having multi-prime-broker relations. However, the
assumed correspondence between buy side and sell side clearing agents is consistent with the CCP clearing arrangements introduced further below and, hence, eases the comparative analysis. Finally, buy side agents are uniformly distributed among sell side prime brokers.

A straight-forward prime broker give-up involves three parties: some sell side agent $i$ being the prime broker of some buy side agent $j$ and some sell side agent $h$ being the executing dealer the buy side agent $i$ trades with. A prime broker give-up results in the decomposition of the original trade in some asset class $l$ between executing dealer $h$ and buy side agent $j$ into two coextensive contracts between buy side agent $j$ and prime broker $i$ and between prime broker $i$ an executing dealer $h$ so that the resulting change in net contract position of $i$ is $\sum_{j=1}^{n} \Delta X_{i,j,l} = 0$ but the net contract positions of $j$ and $h$ with prime broker $i$ change so that $-\Delta X_{j,i,l} = \Delta X_{h,i,l}$. Obviously, a prime broker plays the middle man by being flat back-to-back with regards to market risk. But the prime broker ends up with the dual counterparty risk as a principal to both the executing dealer and the buy side agent.

Generalizing on the above we now transform the original matrix $X_{kbnxk}$ into the matrix $X'_{kbnxk}$ by radically applying multilateral give-ups to all positions in the respective partial buy side matrices and by reflecting the back-to-back nature of give-ups in the new partial sell side matrix by performing the following steps:

1. We reset the new partial sell-side-to-buy-side partial matrix so that $X'_{sbdock} = 0$ and then consolidate all bilateral positions of the buy side agents $j = s + 1, \ldots, s + b$ to a single sell side prime broker agent $p$ each so that $X'_{p,j,l} = \sum_{i=1}^{s} \sum_{l=1}^{k} X_{i,j,l}$.

2. We generate the respective buy-side-to-sell-side matrix by applying the symmetry property so that $X'_{bcoxk} = (-1) \cdot (X'_{xbock})^\tau$.

3. We generate the new sell-side-to-sell-side partial matrix based on the original, i.e. $X'^\tau_{sbdock} = X'^u_{sbdock}$. By fixing the unilateral prime broker relation on $p$ we adjust the new sell-side-to-sell-side matrix to account for the back-to-back relations between prime brokers and executing dealers so that $X'_{p,j,l} = X'_{p,j,l} + X_{i,j,l}$ for $i > p$ and $X'_{p,j,l} = X'_{p,j,l} - X_{i,j,l}$ for $i < p$. Therewith the prime broker agent assumed the
positions the assigned buy side agent built up in trading with other executing dealers on the sell side.

Now, we can calculate the average expected absolute market values of net position exposures for sell side and buy side agents after applying multilateral give-up arrangements denoted by $\gamma$ with:

$$
\varphi_{ss}^\gamma = \frac{1}{s} \left( \sum_{i=1}^{s} \sum_{j=1 \atop j \neq i}^{s} \sum_{k=1}^{k} \sum_{l=1}^{l} \sum_{j=1 \atop j \neq i}^{s} \sum_{k=1}^{k} \sum_{l=1}^{l} \sum_{j=1 \atop j \neq i}^{s} \sum_{k=1}^{k} \sum_{l=1}^{l} X_{i,j,l} \right) = (s-1) \cdot \sqrt{k} \cdot \sqrt{\frac{2}{\pi}} \cdot \sigma_X
$$

(25)

$$
\varphi_{bs}^\gamma = \frac{1}{s} \left( \sum_{i=1}^{s} \sum_{j=1 \atop j \neq i}^{s} \sum_{k=1}^{k} \sum_{l=1}^{l} \sum_{j=1 \atop j \neq i}^{s} \sum_{k=1}^{k} \sum_{l=1}^{l} \sum_{j=1 \atop j \neq i}^{s} \sum_{k=1}^{k} \sum_{l=1}^{l} X_{i,j,l} \right) = b \cdot \sqrt{k} \cdot \sqrt{\frac{2}{\pi}} \cdot \sigma_X
$$

(26)

$$
\varphi_{bs}^\gamma = \frac{1}{b} \left( \sum_{i=1}^{s} \sum_{j=1 \atop j \neq i}^{s} \sum_{k=1}^{k} \sum_{l=1}^{l} \sum_{j=1 \atop j \neq i}^{s} \sum_{k=1}^{k} \sum_{l=1}^{l} \sum_{j=1 \atop j \neq i}^{s} \sum_{k=1}^{k} \sum_{l=1}^{l} X_{i,j,l} \right) = \sqrt{s} \cdot \sqrt{k} \cdot \sqrt{\frac{2}{\pi}} \cdot \sigma_X
$$

(27)

Deriving the changes in $\varphi$ caused by applying multilateral prime broker give-up arrangements delivers:

$$
\tau_{ss}^\gamma = \frac{\varphi_{ss}^\gamma}{\varphi_{ss}^\alpha} = \frac{1}{\sqrt{k}}
$$

(28)

$$
\tau_{bs}^\gamma = \frac{\varphi_{bs}^\gamma}{\varphi_{bs}^\alpha} = \tau_{bs}^\gamma = \frac{\varphi_{bs}^\gamma}{\varphi_{bs}^\alpha} = \frac{1}{\sqrt{k} \cdot s}
$$

(29)

Whilst the netting ratio for the sell-side-to-sell-side matrix does not improve with multilateral prime broker give-up arrangements compared to the close-out netting case, the ratio for the corresponding buy side related matrices does with $\tau_{ss}^\gamma$ and $\tau_{bs}^\gamma$ strictly smaller than the previously derived $\tau_{ss}^\beta$ and $\tau_{bs}^\beta$. Hence, we can formulate the following corollary on the impact of prime broker give-up arrangements:

**Corollary 3:** Prime broker give up arrangements significantly improve the netting efficiency in the buy side segment with the netting ratio increasing in the number of sell side agents participating in such multilateral give-up arrangements times the scope of asset classes covered under such arrangements but with decreasing marginal increments.

As before for close-out netting, the univocal improvement in netting efficiency leaves aside the adverse structural dynamics of prime broker give-up arrangements. For sure, such arrangements create competition for execution on the trading layer among executing dealers.
But downstream on the clearing layer they set all incentives to further concentrate business with just a very few prime brokers.\textsuperscript{17} Hence, the structural dynamics with adverse effects on the distribution of replacement cost and default risk are even more pronounced compared to the previously discussed close-out netting arrangements.

3.6 Multilateral Tear-up Arrangements

In the current market practice multilateral tear-up arrangements and so called portfolio compression services play a role similar to the veteran ring clearing arrangements of early days futures markets. The multilateral offsetting arrangements of the early CBOT clearinghouse founded in 1883 achieved already netting efficiencies of 85-90\% by simply offsetting the bilateral gross exposures and generating a minimal set of bilateral net exposures.\textsuperscript{18} Not till 1925 the CBOT did provide direct default protection through a CCP. In fact, the London Metal Exchange migrated from multilateral offsetting arrangements to a CCP as late as 1986.

Like in the early day ring clearing arrangements, modern day tear-up circles, termination runs and compression services aim at offsetting bilateral trades and portfolio positions as to reduce bilateral gross position exposures. In finding a substituting but minimal set of bilateral positions they ensure that the market value of each agents total net position exposure remains identical to the ingoing net position exposure across the original counterparties. As such, tear-up circles reduce counterparty as well as operational risk.

Although theoretically possible, buy side agents are not participating in large scale tear-up cycles. In our analysis we also assume that tear-ups are restricted to the inner circle of the sell-side-to-sell-side market.\textsuperscript{19} As above for prime broker give-up arrangements we will deploy multilateral tear-up arrangements radically, i.e. all sell side agents make the entire portfolio of positions across asset classes and across all sell side agents subject to the multilateral tear-up arrangements.

\textsuperscript{17} Refer again to Duffie (2009) for a discussion of concentration issues and failure mechanisms of large dealer firms.\textsuperscript{18} Refer to King and Mayer (2009) for a discussion of the competitive dynamics leading to risk dislocations in the prime brokerage sector. Specifically, the authors conclude that to reduce systemic risks, more regulation of prime brokers is warranted to avoid competitive dynamics from undermining counterparty risk management practices.

\textsuperscript{18} See Moser (1998) for details. For the development of historical developments futures markets infrastructures see also Williams (1986) and Kroszner (2006).

\textsuperscript{19} If buy side agents were included in tear-up cycles the offsetting algorithm could result in new buy-side-to-buy-side positions and sell-side-to-buy-side net position exposures. Due to different credit qualities this new allocations would not be favoured by agents with higher credit quality.
For simplicity and without loss of generality we apply a straightforward algorithm which first calculates the total net position exposure across sell side counterparties for each sell side agent and then cascades down from the largest positive and negative (remaining) positions to create a new sell side matrix with the smallest set of bilateral exposures on condition that the total net exposure per sell side agent remains unchanged.\(^{20}\)

We can now transform the matrix \(X' \times_{\text{ss}} \times \times \) into the new matrix \(X'' \times_{\text{ss}} \times \times \) by radically applying multilateral tear-ups per asset class to all positions according to the algorithm introduced above. This results in populating the partial sell-side-to-sell-side matrix \(X'' \times_{\text{ss}} \times \times \) with up to \((s-1) \cdot k\) new bilateral positions \(X'_{i,j,l}\) and \(-X'_{i,j,l}\) on the condition that

\[
\sum_{j=1}^{k} \sum_{l=1}^{s} X'_{i,j,l} = \sum_{j=1}^{k} \sum_{l=1}^{s} X''_{i,j,l} \quad \text{for all } i=1,\ldots,s.
\]

Therewith, the net positions per sell side agent remain unchanged after multilateral tear-up circles.

Since multilateral tear-ups are performed asset class per asset class the resulting allocation of net positions may differ for each asset class. Hence, multilateral tear-ups potentially imply an adverse effect on the efficiency of close-out netting arrangements. Hence, the average, across sell side agents, expected absolute market values of net position exposures for sell side agents after applying multilateral tear-ups is given by the average of fully retaining the close-out advantage, denoted by \(\Phi^{-\delta}_{\text{ss}}\), vs. fully losing the close-out advantage, denoted by \(\Phi^{+\delta}_{\text{ss}}\). The lowest possible average position exposure \(\Phi^{-\delta}_{\text{ss}}\) and the highest possible average position exposure \(\Phi^{+\delta}_{\text{ss}}\) are defined as

\[
\Phi^{-\delta}_{\text{ss}} = \frac{1}{s} \left( \sum_{i=1}^{s} \left( \sum_{j=1}^{k} \sum_{l=1}^{s} X''_{i,j,l} \right) \right) = \sqrt{k} \cdot \sqrt{s} \cdot 1 \cdot \frac{\sqrt{2}}{\sqrt{\pi}} \cdot \sigma_X \tag{30}
\]

\[
\Phi^{+\delta}_{\text{ss}} = \frac{1}{s} \left( \sum_{i=1}^{s} \left( \sum_{j=1}^{k} \sum_{l=1}^{s} -X''_{i,j,l} \right) \right) = k \cdot \sqrt{s} \cdot 1 \cdot \frac{\sqrt{2}}{\sqrt{\pi}} \cdot \sigma_X \tag{31}
\]

Hence, the expected average position exposure is defined as

\[
\Phi^{\delta}_{\text{ss}} = \frac{\Phi^{+\delta}_{\text{ss}} + \Phi^{-\delta}_{\text{ss}}}{2} = \sqrt{s} \cdot 1 \cdot \frac{\sqrt{2}}{\sqrt{\pi}} \cdot \frac{k + \sqrt{k}}{2}
\]

\(^{20}\) The algorithm applied here is identical to the one introduced by Jackson and Manning (2007).
Consequently, the respective netting ratio improves to:

\[ \tau_{ss}^\delta = \frac{\varphi_{ss}^\delta}{\varphi_{ss}^\gamma} = \frac{1}{2\sqrt{s} - 1} \left( 1 + \frac{1}{\sqrt{k}} \right) \]  

(33)

All buy side related matrices remain unchanged. So do the average position exposures and netting ratios, i.e. \( \tau_{ss}^\delta = \tau_{ss}^\gamma \) and \( \tau_{bs}^\delta = \tau_{bs}^\gamma \). But for the sell-side-to-sell-side matrix the netting efficiency has improved because of \( \tau_{ss}^\delta < \tau_{ss}^\gamma \) for all reasonable combinations of \( k \) and \( s \).\(^{21}\) Hence, we can formulate the following corollary with respect to multilateral tear-up arrangements:

**Corollary 4:** Tear-up arrangements always improve the netting efficiency in the participating sell-side-to-sell-side market segment. Because multilateral tear-ups are performed per asset class some of the efficiency gains of close-out netting will be lost on average. But the overall efficiency gains are owed to the netting across sell side counterparts where the netting ratio naturally increases in the number of sell side agents participating but with decreasing marginal increments. Buy side related market segments remain unaffected.

It is the root of the matter of multilateral offsetting arrangements that by reducing the initial clutter to a minimum of bilateral exposures they increase concentration of replacement cost and default risk. Whilst there seems no direct impulse for further concentration, economies of scale and scope will play in favour of large dealers. Again, it remains on the agenda for our numerical simulations to analyse the distributional effects of multilateral tear-up arrangements.

### 3.7 CCP Clearing Arrangements

We touched already on the subject of the historical evolution of clearing arrangements with the CBOT example. The introduction of a CCP for to replace the multilateral ring clearing with a new regime providing direct protection of members appears as a natural advance in delivering market integrity, broadening the member base and, ultimately, boosting liquidity. In fact, in terms of the netting scheme there is only a little difference between multilateral ring and CCP clearing arrangements; this little but decisive difference being the novation of trades to the CCP as the ultimate risk taker. A CCP interposes itself as the principal counterparty of

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\(^{21}\) The inequality \( \tau_{ss}^\delta < \tau_{ss}^\gamma \) is satisfied when \( s > \left( \frac{\sqrt{k} + 1}{2} \right)^2 - 1 \). For example, it needs just 2 sell side prime brokers to satisfy the condition for up to 6 asset classes.
record for all trades, i.e. the CCP becomes the seller to every buyer and the buyer to every seller. Hence, a CCP protects members from replacement cost risk arising from another member’s default.

In essence, a CCP is a multilateral venue to mutually share replacement cost risk among a group of clearing members. Beside rigid margining regimes and enforceable default resolution protocols the major risk mitigation tools of a CCP are its tiered, clearing member centric structure and its risk capital structure, i.e. its horizontal and vertical risk sharing structure.

(1) **Horizontal Risk Sharing Structure**: Only clearing members have a direct principal relationship with the CCP. All trades and resulting positions of non-clearing members and customers are guaranteed by the assigned clearing member. There is usually a one-to-one correspondence between clearing and non-clearing members. The clearing member is liable for any liabilities and open positions; being it those of customers or those of its own. The CCP ensures the integrity of its member base by capital adequacy requirements and ongoing credit monitoring.

Following a default, the CCP assumes any net unsettled obligations and positions from the defaulting clearing member including those of the clearing member’s segregated customer accounts. Usually customer accounts are transferred to non-defaulting clearing members rather than being closed by force as it is done with the non-segregated proprietary accounts of the defaulting clearing member. Consequently, in the transition from a non-CCP cleared to a CCP cleared market, buy side agents gain because the CCP – or better all members mutually contributing to the risk capital of the CCP – insure buy side agents against the default of their clearing members.

(2) **Vertical Risk Sharing Structure**: Following a member default – or the defaults of multiple members – a CCP attempts to close-out unsettled obligations and open contract positions of the non-segregated proprietary accounts of the defaulting members in a timely and non-destabilizing manner. To cover any potential replacement cost resulting from that liquidation or any other obligations the defaulting

---

22 For CCP where the legislative provisions allow for it, default resolution procedures stipulate that segregated customer accounts including the assigned customer collaterals are transferred to other non-defaulting clearing members. As the default handling of Lehman has shown, the ease and promptness with which segregated customer accounts can be transferred helps preserve market confidence and integrity in times of stress. See also EACH (2009).

23 See also Pirrong (2009) for a more comprehensive discussion of redistributive effects of risk-sharing through a central clearing mechanism.
members fails to perform on, the CCP can draw from an order of clearing member, shareholder and external resources – collectively referred to as “lines of defence”. For virtually all large-scale CCP the clearing default fund is the primary “line of defence”. Clearing members are required to make initial and replenishing contributions to fill the clearing default fund. On this note, the CCP “socializes” default risks by means of its clearing default fund.\textsuperscript{24}

We gradually introduce CCP clearing arrangements by first limiting the scope to the sell side and later expand the analysis to cover buy side clearing. This allows us to firstly expose the impact of splitting up the clearing of some asset classes through a CCP while the other asset classes remain in the prevailing multilateral clearing structure. As a second step we abstract away from splitting up asset classes but rather focus on showing the impact of migrating the buy side to a CCP clearing.

In general, CCP clearing is added to the original matrix representation by adding some further partial matrices representing the market values of net positions the CCP has with sell side agents acting as clearing members and vice versa. Hence, the expanded matrix is defined as the third instance of our original representation as

\[
X''_{(n+1)x(n+1)xk} = \begin{pmatrix}
X''_{sxo\times k} & X''_{sxb\times k} & X''_{sdb\times k} \\
X''_{b xo\times k} & X''_{b xo\times k} & X''_{b db\times k} \\
X''_{b xo\times k} & X''_{b xo\times k} & X''_{b db\times k}
\end{pmatrix}
\]

(34)

The new matrix inherits all properties of the prior matrices. We take the matrix \(X'_{no\times ok}\) after close-out netting and multilateral prime broker give-ups but before multilateral tear-ups as the basis, because with respect to consolidating buy side positions onto sell side agents there is no structural difference between prime brokerage and clearing member centric clearing arrangements. We fill the new matrix by simply copying the partial matrices \(X'_{sxo\times k}\), \(X'_{sxb\times k}\), \(X'_{b xo\times k}\), and \(X'_{b db\times k}\) to their congruent counters of \(X''_{(n+1)x(n+1)xk}\). According to the consideration on the member-centric structure with indirect access only for buy side agents, we set the buy-side-to-CCP and the CCP-to-buy-side matrices to zero. We also set the CCP-to-CCP partial matrix zero.

\textsuperscript{24} In addition, some CCP have contingent risk capital available to cover overshooting losses not absorbed by the clearing default fund. Contingent capital includes assessment rights on members, guarantees by third parties like banks and re-insurance companies.
We start with the strong claim that all asset classes are subject to CCP clearing arrangements and that all sell side agents are forced to participate, i.e. we consider an exclusive single CCP clearing arrangements. At this point, we do not yet consider any participation of buy side agents in the CCP clearing arrangements.

We populate the sell-side-to-CCP matrix so that for all sell side agents the positions are netted across sell side counterparts and moved to the non-segregated proprietary accounts held with the CCP, i.e. $X^\sigma_{i,n+1,l} = \sum_{j=1}^{k} X^\sigma_{i,j,l}$ for all $i=1,\ldots,s$. To conclude the novation to the CCP we set $X^\sigma_{ssck}$ to zero and create the corresponding CCP-to-sell-side partial matrix by setting $X^\sigma_{ccks} = (-1) \cdot \left(X^\sigma_{ssck}\right)^T$.

We first derive the average absolute position exposures for the theoretically feasible but less realistic case of a mandatory CCP clearing for all asset classes in a single CCP:

$$
\varphi_{x(s+c)}^x = \frac{1}{s} \left( \sum_{j=1}^{k} E \left( \sum_{i=1}^{s} X^\sigma_{i,n+1,l} \right) \right) = \sqrt{k} \cdot \sqrt{s-1} \cdot \sqrt{\frac{2}{\pi}} \cdot \sigma_X
$$

Calculation of the corresponding netting ratio results in:

$$
\tau_{x(s+c)}^x = \frac{\varphi_{x(s+c)}^x}{\varphi_{ssx}^x} = \frac{1}{\sqrt{k \cdot (s-1)}}
$$

Since we excluded buy side clearing so far, all buy side related matrices remain unchanged and so do the average absolute position exposures and netting ratios, i.e. $\tau_{ssb}^x = \tau_{ssb}^y$ and $\tau_{bsx}^x = \tau_{bsx}^y$. With the netting ratio for exclusive CCP clearing in the sell-to-sell-side segment being strictly smaller than the ratio with multilateral tear-ups under the condition that there is more than one asset class, i.e. $\tau_{ssx}^x < \tau_{ssx}^y$ for $k > 1$, we can formulate the following corollary:

**Corollary 5:** In a sell side only exclusive single CCP scenario for all asset classes, CCP clearing would maximize the netting efficiency in the sell side segment whilst the netting efficiency of buy side related market segments would remain unaffected. Only for the hypothetical case of a single asset class universe multi-lateral tear-ups could lead to netting ratios comparable or equal to the ratio derived for the exclusive single CCP clearing scenario.
To derive a more realistic scenario, we split the $k$ asset classes into $z$ non-centrally-cleared asset classes and $(k-z)$ centrally-cleared asset classes. We reset $X_{ssock}^\sim$ to $X_{ssock}^\sim = X_{ssock}'$. Then we modify the matrices $X_{csock}^\sim$ and $X_{rsock}^\sim$ so that the split is reflected by satisfying the condition $X_{i,n+1,l}^\sim = \sum_{j=1}^{l} X_{i,j,l}^\sim$ across all $i=1,...,s$ but only for the $l=1,...,k-z$ centrally-cleared asset classes. Accordingly we set only the subset of $X_{csock}^\sim$ with $l=1,...,k-z$ to zero and leave the remaining of $X_{csock}^\sim$ unchanged. The transformation $X_{rsock}^\sim = (-1) \cdot (X_{csock}^\sim)^\top$ concludes the novation of all net position exposures in $(k-z)$ asset classes to the single CCP.

Assuming that the remaining $z$ asset classes will be subject to multilateral tear-ups, i.e. the remaining net position exposures of $X_{rsock}^\sim$ will be changed accordingly, we derive the average absolute net position exposure for some sell side agent by adding up the respective results for the non-centrally-cleared asset classes and the centrally-cleared asset classes. The result for the non-centrally-cleared assets classes $\phi_{xss(i+c)}^z$ subject to multilateral tear-ups replicates the result in (32) with $k$ replaced by $z$, whereas the result for centrally-cleared asset classes $\phi_{xsc(i+c)}^z$ replicates the result in (35) with $k$ replaced by $(k-z)$. By adding the average absolute exposures in cleared and non-cleared asset classes we get:

$$\phi_{x,s(i+c)}^{z,k} = \phi_{x,s(i+c)}^{z,k-z} + \phi_{x,s(i+c)}^{z,z} = \sqrt{s-1} \cdot \left( \sqrt{k-z} + \frac{z + \sqrt{z}}{2} \right) \cdot \sqrt{\frac{2}{\pi} \cdot \sigma_X} \quad (37)$$

The corresponding netting ratio is given by:

$$\tau_{x,s(i+c)}^{z,k} = \frac{\phi_{x,s(i+c)}^{z,k}}{\phi_{x,s}^{z}} = \frac{1}{\sqrt{k(s-1)}} \cdot \left( \sqrt{\frac{k-z}{k}} + \frac{z + \sqrt{z}}{2\sqrt{k}} \right) \quad (38)$$

Obviously, there is an adverse effect on the netting efficiency achieved in the exclusive single CCP clearing case. Except for the extreme case where $z=0$ and the term in brackets in (38) equals 1, i.e. when and all asset classes are centrally-cleared, the term is always greater than 1 and is increasing in $z$. The scale of the term depends on the ratio of cleared assets to total assets, i.e. the first summand of the term in brackets, and on the effectiveness of the close-out netting under multilateral tear-up arrangements for the non-centrally-cleared asset classes, i.e. the second summand of the term. As with the exclusive single CCP clearing the netting efficiency in the buy side related segments remains unaffected.
Comparing the combined netting ratio $\tau^{\lambda;\gamma;k} \left( \frac{\lambda}{\gamma} \right)_{x_0(x+c)}$ with the ratio $\tau^\delta \left( \frac{\delta}{\gamma} \right)_{x_0(x+c)}$ of multilateral tear-ups only, it becomes obvious that the inequality $\tau^{\lambda;\gamma;k} \left( \frac{\lambda}{\gamma} \right)_{x_0(x+c)} < \tau^\delta \left( \frac{\delta}{\gamma} \right)_{x_0(x+c)}$ does not always hold. However, for a reasonable total number of asset classes $k$, e.g. for all $k \leq 10$, the tipping point in favour of the combined CCP clearing scenario would be achieved if more than 2 asset classes would be cleared centrally. Simplified, one could conclude that for the 6 OTC derivatives asset classes as reported on a regular basis in BIS (2009) more than 2 should be cleared by a CPP to create a clear cut advantage for the combined CCP clearing scenario. Taking the relative size in gross market values of the particular OTC derivatives asset classes as a basis, even a lower number of cleared asset classes could do. The relative weighting of such lower number of asset classes would have to be roughly above one-third of the total. For example, CCP clearing of Interest Rate Swaps (IRS) counting for roughly 60% of total market values could already do the trick. Clearing of Credit Default Swaps (CDS) alone will most probably not hit the tipping point because they count for hardly 11% of total market values.  

Summarizing, we can formulate the following corollary on the co-existence of CCP clearing and multilateral OTC clearing arrangements:

**Corollary 6:** By lifting exclusivity and allowing the split into cleared and non-cleared asset classes in the sell side segment the netting efficiency always suffers in comparison to an exclusive CCP clearing scenario. The netting advantage of this combined CCP clearing scenario over the multilateral tear-ups only scenario is given if a certain minimum threshold of centrally-cleared asset classes is exceeded. Ceteris paribus, the netting efficiency of buy side related market segments remains unaffected.

We conclude our comparative analysis by including the buy side in the CCP clearing arrangements. We do so along the lines of the introducing remarks on the member centric structure of a CCP made at the beginning of subchapter. We abstract away again from the splitting in centrally-cleared and non-centrally-cleared asset classes. This allows for solely exposing the impact of buy side CCP clearing. We take the exclusive single CCP clearing case as the basis for the considerations on the buy side CCP clearing.

In principle, participation of buy side agents as non-clearing members in CCP clearing arrangements changes only a small feature of the netting mechanics. Under the prime brokerage umbrella position exposures of buy side clients and executing sell side dealers are

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25 Duffie and Zhu (2009) arrive at similar conclusions when comparing an extreme setup without any multilateral clearing arrangements like give-ups and tear-ups with a CCP setup.
intermediated back-to-back by the sell side prime broker agents as principal counterparty. In this context, the prime broker agent was able to net out the positions assumed for the back-to-back trades. To the contrary, most CCP do not allow for netting across customer positions passed on to the CCP. Hence, a sell side agent acting as clearing member for some buy side customers is supposed to keep the customer positions in segregated accounts and the CCP does not net across these segregated accounts. As a consequence, the sell side agent acting as a clearing member for some buy side agents will have additional position exposures vis-à-vis the CCP in maintaining segregated customer accounts.

Assuming that buy side clients are equally distributed among the sell side agents acting as clearing members there are on average \( \frac{b}{s} \) buy side agents clearing through one sell side agent. Denoted with \( w \) we derive the average of the absolute position exposure with CCP of a sell side agent clearing for buy side agents by adding the previously derived average absolute position exposure of a buy side agent \( g_j^{\text{sb}} \) multiplied by the average number of buy side agents to the already known average absolute net position exposure for the non-segregated account \( l_j^{\text{css}} \). This delivers:

\[
\phi_{\text{sb}}^{\lambda} = g_j^{\text{sb}} \cdot w + l_j^{\text{css}} \cdot \left( \sqrt{k \cdot (s-1)} + \frac{b}{s} \cdot \sqrt{k \cdot s} \right), \sqrt{\frac{2}{\pi}} \cdot \sigma_X
\] (39)

Accordingly, the netting ratio is defined as:

\[
\tau_{\text{sb}(x+c)}^{\text{b}} = \frac{\phi_{\text{sb}(x+c)}^{\text{b}}}{\phi_{\text{sb}(x+c)}^{\lambda}} = \frac{1}{\sqrt{k \cdot (s-1)}} + \frac{b}{(s-1) \cdot \sqrt{k \cdot s}}
\] (40)

Apparently, there is an adverse effect on the netting ratio achieved in (36) by sell side only clearing. It always holds that \( \tau_{\text{sb}(x+c)}^{\text{b}} > \tau_{\text{sb}(x+c)}^{x} \) because buy side clearing puts an additional burden on sell side clearing agents in maintaining segregated position and collateral accounts for their customer business with the CCP. They virtually have to take all back-to-back contracts created by trades of their buy side customers off their non-segregated account with the CCP and transfer them on a gross basis to the segregated accounts held with the CCP for customer business. This scaling back of the netting efficiency is exposed in the second summand in (40). Hence, the adverse effect increases in the number of buy side agents \( b \) participating in the clearing as non-clearing members whilst it is cushioned by the diversification effect agents achieve in consolidating positions across \( (s-1) \) executing dealers.
to a single sell side agent. This netting effect is expressed by the reciprocal of \((s - 1) \cdot \sqrt{k \cdot s}\) and follows from (29) above.

From a buy side agent’s perspective there is no difference in terms of average absolute net position exposure nor in terms of netting efficiency, i.e. \(\tau^{0}_{bsb} = \tau^{x}_{bsb}\) and \(\tau^{0}_{cbs} = \tau^{x}_{cbs}\). But, as already noted above, there is a difference in terms of the actual risk exposure because the CCP guarantees the buy side agent’s position in case of a default of the clearing agent. The strict segregation of positions and corresponding collaterals makes customer portfolios fungible in case of default of the sell side clearing agent.\(^{26}\) In this sense, sell side clearing members of a CCP effectively insure the buy side customers against default risk. In essence, buy side agents are major beneficiaries of CCP clearing because their default risk exposure changes from being exposed to the sell side dealer to being exposed to the much more diversified and lower default risk of the CCP. As a result, buy side agents may increase their trading activity and, in doing so, may further increase the risk taking of the CCP and the sell side agents backing the CCP, respectively.

At this point, we arrive at the final corollary on CCP clearing for sell side and buy side agent:

**Corollary 7:** Including the buy side in CCP clearing arrangements under a member centric clearing structure creates, ceteris paribus, an adverse effect on the netting ratio in the sell side segment caused by the common requirement of CCP to segregate customer position and collateral accounts. Whilst the netting efficiency of buy side related market segments remains unaffected, the buy side agents are major beneficiaries because their positions exposures are mutually ensured by all sell side agents backing the CCP.

By its nature, a CCP changes the distribution of replacement cost and default risk in the market. Contrary to the potential implications challenged for all previous clearing arrangements, CCP clearing distributes rather than concentrates default risk among its sell side clearing members; the only unidirectional change being the transfer of risk away from buy side agents. Buy side agents gain insurance as non-clearing members by the CCP against default of their chosen sell side clearing agent.

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\(^{26}\) In the aftermath of the Lehman bankruptcy, segregated asset and collateral accounts are a highly desired feature demanded by buy side customers from prime brokers. The increased credit concerns vis-à-vis prime brokers and dealers led to an increase in multi-prime-broker. See also the note on re-hypothecation issues above and Singh and Aitken (2009).
4 SUMMARY OF FINDINGS AND RESEARCH OUTLOOK

4.1 Initial Findings on Netting Efficiency

In narrowing down the comparative analysis of clearing arrangements to our initial metrics based on netting efficiency it seems that one could square the circle by just combining all of the analysed arrangements. The comparison of average absolute position exposures and netting ratios as shown in Table 1 exposes a strict and univocal order. Each of the analysed arrangements improves over the others at least in one of the sell side and buy side market segments.

Within our static framework and against our metrics for comparison CCP clearing appears to be the silver bullet to maximize netting efficiency and reduce, ceteris paribus, average replacement cost and average collateral cost to a minimum. The only caveat being, that if CCP clearing is not *exclusive*, the introduction of a CCP may distract synergies and lower the overall netting efficiency if only a small fraction of the market is cleared via the CCP. There seems to be good reason to assume that a certain portion of asset classes – or rather a certain share of less standardized and illiquid contracts within an asset class – remains out of scope for CCP clearing.

Against the background of the size of OTC derivatives positions held by large sell side dealers, the increasing netting efficiency – expressed in the decreasing average absolute net position per sell side dealer and netting ratio per market segment – reduces replacement cost risk, and therefore reduces the losses that surviving agents suffer as a result of the default of some agent. From this it follows, prima facie, that maximizing netting efficiency reduces the likelihood that the failure of one of the systemically critical large sell side dealer will cause the failure of others, i.e. that maximizing netting efficiency is a means to reduce systemic risk.
Table 1: Summary of comparative analysis

<table>
<thead>
<tr>
<th></th>
<th>( \alpha )</th>
<th>( \beta )</th>
<th>( \gamma )</th>
<th>( \delta )</th>
<th>( \lambda )</th>
<th>( \kappa )</th>
<th>( \omega )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Initial reference matrix</td>
<td>Close-out netting</td>
<td>Prime broker give-ups</td>
<td>Multilateral tear-ups</td>
<td>Combined CCP Clearing</td>
<td>Exclusive CCP Clearing</td>
<td>Exclusive CCP Clearing</td>
</tr>
<tr>
<td></td>
<td>( (s-1) \cdot k )</td>
<td>( (s-1) \cdot \sqrt{k} )</td>
<td>( (s-1) \cdot \sqrt{k} )</td>
<td>( \sqrt{s-1} \cdot \frac{k+\sqrt{k}}{2} )</td>
<td>( \sqrt{s-1} \cdot \left( \sqrt{k-z} + \frac{\sqrt{z}}{2} \right) )</td>
<td>( \sqrt{k} \cdot (s-1) )</td>
<td>( \sqrt{k} \cdot (s-1) + \frac{b}{s} \cdot \sqrt{k \cdot s} )</td>
</tr>
<tr>
<td>( \varphi_{ss,s} )</td>
<td>( \varphi_{ss,(s+c)} )</td>
<td>( \tau_{ss,s} )</td>
<td>( \tau_{ss,(s+c)} )</td>
<td>( b \cdot k )</td>
<td>( b \cdot \sqrt{k} )</td>
<td>( b \cdot \sqrt{k} )</td>
<td>( b \cdot \frac{k}{\sqrt{s}} )</td>
</tr>
<tr>
<td>( \varphi_{ss,b} )</td>
<td>( \tau_{ss,b} )</td>
<td>( \varphi_{bs,s} )</td>
<td>( \tau_{bs,s} )</td>
<td>( s \cdot k )</td>
<td>( s \cdot \sqrt{k} )</td>
<td>( s \cdot \sqrt{k} )</td>
<td>( s \cdot \sqrt{k} )</td>
</tr>
</tbody>
</table>

\( \varphi_{ss} \): average multiplier, across sell side, of absolute position exposure of a sell side agent; \( \tau_{ss} \): netting ratio in sell-side-to-sell-side segment; \( \varphi_{ss,b} \): average multiplier, across buy side, of absolute position exposure of a sell side agent; \( \tau_{ss,b} \): netting ratio in sell-side-to-sell-side segment; \( \varphi_{bs,s} \): average multiplier, across buy side, of absolute position exposure of a buy side agent; \( \tau_{bs,s} \): netting ratio in sell-side-to-sell-side segment; \( s \): number of sell side agents; \( b \): number of buy side agents; \( k \): number of asset classes; \( z \): number of non-cleared asset classes.
4.2 Research Outlook

The comparative analysis within the comprehensive but static framework developed in this paper aimed at comparing the netting efficiency and average replacement cost under different clearing arrangements. Our focus was rather on an end-to-end representation of the organization of the market with particular emphasize on the tiered market structure and the central role of sell side agents, which act as prime brokers, clearing members and liquidity providers. For the purpose of laying a sound foundation with framework we abstracted away from representing and analysing structural dynamics and distributional effects on the risk profile of agents and the market as a whole. We also placed very limiting assumptions on the distribution of trading positions and risk factors. We also compromised on a very simplified margining regime and agent representation by not allowing for differing credit qualities and default probabilities. However, the framework developed is a sound basis for challenging these points. To begin with, the framework should be amended to relax some of the assumptions made in the initial model and to cope with the following aspects:

(1) **Prime Broker/Clearing Member Assignments:** Dynamics in position assignment to prime brokers by buy side clients and alternative loss-sharing schemes within a CCP should be considered. Furthermore, we will address the question of single versus multiple CCP including the option to link CCP.

(2) **Homogenous Agents Assumption:** In the initial model all agents have the same ex-ante default probabilities. The sell and buy side differentiation is based solely on different trading patterns resulting from existing market structure. Relaxation of the agent homogeneity assumption will allow for tailoring the margin requirements to the market participants’ credit quality, as observed in the OTC markets.

(3) **Exogenously Defined Default Probabilities:** Exogenous default implies that an agent’s market risk and credit risk are independent; i.e. default probabilities are independent of position and market price changes. In practice, the adverse market movements and extreme tilted position profiles substantially increase default probabilities as well as loss given default ratios. Therefore, balance sheet risk of agents should be incorporated into the model as a further variable influencing default probability, and default probability itself should be considered as an endogenous variable.

(4) **Normal Distribution Assumption:** In the model at hand, the normal distribution of the market values of the net contract positions and of the changes in market values across asset classes is assumed. Further, it is assumed that changes in positions and changes
in underlying risk factors are independent. In reality, distributions are both, more skewed as well as correlated.

Various issues regarding potential dislocations in the distribution of replacement cost risk and structural imbalances in market activity surfaced already when deriving our initial findings. To gain a more comprehensive understanding of the versatile implications of the alternative clearing arrangements we plan to use numerical simulations in future research. This should allow for an in depth consideration in regard to concentration and systemic risk.

The financial crisis has highlighted the need for a deeper understanding of systemic risk and externalities in financial networks and shock transmission processes at the systemic level. An analysis of structural dynamics and risk concentration under alternative clearing arrangements will contribute to a better insight into the implications of the propagated market reforms.
REFERENCES


