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Real Exchange Rates and Misalignments

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Abstract: This paper examines the interaction of G7 real exchange rates with real output and interest rate differentials. Using cointegration methods, we generally find a link between the real exchange rate and the real interest differential. This finding contrasts with the majority of the extant research on the real exchange rate – real interest rate link. We identify a new measure of the equilibrium exchange rate in terms of the permanent component of the real exchange rate that is consistent with the dynamic equilibrium given by the cointegration relation. Furthermore, the presence of cointegration also allows us to identify real, nominal and transitory disturbances with only minimal identifying restrictions. Our findings suggest that persistent deviations of real exchange rates from their equilibrium value can have feedback effects on the underlying fundamentals, hence altering the equilibrium exchange rate itself. This has important implications for the persistence measures of real exchange rates that are reported elsewhere in the literature.

Keywords: Equilibrium Exchange Rates; Cointegration; Permanent and Transitory Decomposition.

JEL classification: F31

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1 Introduction

For a number of reasons related to the design of macroeconomic policy, identifying the sources of real exchange rate fluctuations ranks high in the research agenda on international finance. For example, if real exchange rates are driven primarily by real factors, then a flexible exchange rate regime may be required to allow economies to adjust to relative shocks. If, however, nominal fluctuations - and here more particularly cross-country variations in monetary policy - are responsible for the observed behaviour, then (and to the extent that exchange rate variability is perceived as a bad) monetary policy coordination, or even the fixation of nominal exchange rates, may be the better choice. Given the choice of a particular exchange rate regime, knowledge of the sources of shocks to the real exchange rate may be of use to a policy maker interested in assessing if a currency is misaligned or not.

Starting with Clarida and Gali (1994), a number of papers have focussed on the importance of business cycle shocks - the sum of demand and nominal - relative to supply side shocks in explaining real exchange rate movements. Practically all of this recent literature has ignored a variable which is pivotal to most exchange rate models, namely the real interest differential. Furthermore, the common practice of using a vector autoregressive model in first differences means that any potential cointegrating relationships amongst the variables driving the shocks are ignored. However, it is conceivable that the introduction of the levels of the variables has implications for the findings reported by others. In this paper, we model the interaction of G7 real exchange rates with real output and real interest rate differentials. Using cointegrated vector autoregressions, we find a long-term link between real differentials and real exchange rates.

The setup of our econometric model also allows us to relate to the empirical literature on exchange rate misalignment (for a survey see MacDonald (2000)). We define the equilibrium real exchange rate as the one that is consistent with the dynamic equilibrium that is represented by the cointegrating relationship found in the data. We compare this cointegration-based notion of an equilibrium exchange rate to the notion that has been proposed by Clarida and Gali (1994).

The cointegration based method delivers measures of misalignment that are highly correlated with the Clarida-Gali measure but that are considerably less persistent. We show that this may be due to the fact that the cointegration based method allows for dynamic feedbacks of a misaligned exchange rate onto the fundamentals. Our results may have important implications for the discussion about the excess persistence of exchange rates, i.e. the exchange rate puzzle.
Our results also seem to build a bridge between some conflicting results in the literature. For example, earlier contributions (e.g. Clarida and Gali (1994), Weber (1997) and Rogers (1998)) tended to interpret real shocks as permanent and nominal shocks as transitory and work with a first difference representation of a VAR. Depending on the information set used, these contributions would either emphasize the role of nominal disturbances (such as Clarida and Gali (1994), Rogers (1999)) or find a very important role for permanent disturbances (such as Weber (1997)). Our approach identifies permanent and transitory components based only on the cointegrating information in the data. We then disentangle real and nominal permanent shocks using the approach pioneered by Blanchard and Quah. We find that the bulk of real exchange rate variation is explained by permanent shocks but that, on average, in the G7, both real and nominal forces play an important role.

The remainder of the paper is structured as follows. In section two we provide a review of the literature that aims to explain exchange rate persistence and volatility. In section three we provide a motivational model for our empirical analysis based on Clarida and Gali (1994). In section four, our econometric model and empirical results are presented and section five concludes.

2 Exchange Rate Persistence and Volatility: A Motivational Overview.

In this section we present a brief overview of some of the key empirical results concerning the persistence and variability of real exchange rates which have a bearing on the empirical results presented in section 4. The purpose of this review is to motivate the model considered in the next section and also our empirical results. A useful starting point is to define the real exchange rate, $q_t$, as:

$$q_t = s_t - p_t + p_t^*$$

where $s_t$ denotes the nominal exchange rate (home currency price of a unit of foreign exchange), $p_t$ denotes a price level, an asterisk denotes a foreign magnitude and lower case letters denote that a logarithmic transformation has been used. A proponent of strict PPP would argue that the real exchange rate should always equal zero, although the use of price indices to calculate $q_t$, or the existence of constant transaction costs, means that it may hold up to a constant term. However, using univariate unit root methods and data for the recent floating period a number of researchers find that real exchange rates are effectively unit root process and do not exhibit any significant mean reversion (see Mac-
Donald (1995)).\textsuperscript{1} However, a significant transitory, or mean reversion, component is reported recovered from long time span data sets (see, for example, Edison (1987), Frankel (1986,1988), Abuf and Jorion (1990), Grilli and Kaminsky (1991) and Diebold, Husted and Rush (1991)). Using around 100 years of annual data these studies find that half of a shock to the real exchange rate is offset after four years. A similar half-life is reported in studies which exploit panel data sets for the recent floating period, (see, for example, Bayoumi and MacDonald (1999), Frankel and Rose (1995), Wu (1995), Oh (1995) and MacDonald (1995b)).

How may the above result be explained? One explanation, which is in the spirit of a traditional PPP, involves recognizing the implications that non-zero transaction costs can have for the time series properties of real exchange rates. For example, a number of theoretical papers (see, for example, Dumas (1992) and Ser cu, Uppal and Van Hulle (1995)) have demonstrated that if markets are spatially separate, and feature proportional transactions costs, deviations from PPP should follow a non-linear mean-reverting process, with the speed of mean reversion depending on the magnitude of the deviation from PPP. The upshot of this is that within the transaction band deviations from PPP are long-lived and take a considerable time to mean-revert: the real exchange rate is observationally equivalent to a random walk. However, large deviations - those that occur outside the band - will be rapidly extinguished and, for them, the observed mean reversion should be very rapid. The existence of other factors, such as the uncertainty of the permanence of the shock and the so-called sunk costs of the activity of arbitrage may widen the bands over-and-above that associated with simple trade restrictions (see Dixit (1989) and Krugman (1989)). A number of papers (see, inter alia, Obstfeld and Taylor (1997), Michael, Nobay and Peel (1997), O’Connell (1996) and O’Connell and Wei (1997)) have implemented this idea using band threshold autoregressive models and find support for the non-linear hypothesis. For example, Obstfeld and Taylor use a band threshold autoregressive model to estimate mean reversion speeds for real exchange rates, defined using both CPI and disaggregate price series. For the CPI-based real exchange rates they report adjustment speeds outside the transaction band of one year, while for the disaggregate prices they report adjustment speeds as low as 2 months.

An alternative way of reconciling these findings is to consider the pricing-to-market and pass-through models that have received increased attention in the recent so-called ‘new open-economy’ macroeconomics

\textsuperscript{1}Studies which use the variance ratio statistic to test for mean reversion in real exchange rates come to a similar conclusion (see, for example, Huizinga (1987), Glen (1992) and MacDonald (1995)).
(see Lane (1999) for a survey). In traditional Keynesian models, prices are sticky in terms of domestic currencies. Hence, fluctuations in the nominal exchange rate alter the competitiveness of the domestic export sector and redirect foreign demand towards domestic goods - the so-called expenditure-switching effect. As a consequence, countries experiencing shocks to the real exchange rate that produce a depreciation should run trade balance surpluses. Recent models, have relaxed the Keynesian pricing assumption, introducing alternative price-setting schemes such as pricing-to-market (PTM) and Local-currency-pricing (LCP) (see Devereux (1997)).

Under the pricing to market-scheme, producers engage in price discrimination by setting different markups over marginal costs in domestic and foreign markets. Under the traditional Keynesian pricing scheme, exchange-rate changes pass through immediately to the foreign market, the law of one price holds. With PTM, however, exporters will set the same mark-ups ex-ante but shocks occurring after price-setting, combined with the effective separation of home and foreign markets will generally lead to only partial pass-through and to deviations from the law of one price. Under local-currency pricing, foreign producers set prices on the home market in domestic currency. Hence, there is no exchange-rate pass-through at all. Recent models have used PTM-cum-LCP setups and can successfully explain the high variability of both real and nominal exchange rates as well as international consumption correlations. In recent work, however, Obstfeld and Rogoff (1999) cast doubt on the empirical relevance of the PTM-LCP assumption, in spite of its ability to help in rationalizing key facts of international macroeconomic fluctuations. One reason for this is to be found in the work of Rogers and Jenkins (1995) and Wei and Parsley (1995). These researchers show that adjustment speeds for disaggregate prices are similar to the adjustment speeds found for aggregate CPI real exchange rates and this seems inconsistent, at least, with the PTM story since it would imply that their is a one-to-one relationship between the firms pricing policy and the exchange rate.

An alternative explanation for real exchange rate persistence involves recognizing that there are real determinants of real exchange rates, such as net foreign assets and Balassa-Samuelson productivity effects. Often these kinds of effects are modelled using the real interest rate parity condition:

$$\Delta q_{t+k} = -(r_t - r_t^*)$$, \hspace{1cm} (2)

Since this approach is normally applied to real effective exchange rates, the real exchange rate is expressed as the foreign currency price of a unit
of home currency. Expression (2) may be rearranged as an expression for the real exchange rate as:

\[ q_t = q_{t+k} + (r_t - r_t^*) \]  

(3)

and if \( q_{t+k} \) is interpreted as the 'long-run' or systematic component of the real exchange rate, \( \tilde{q}_t \), we may re-write this as:

\[ q_t = \tilde{q}_t + (r_t - r_t^*) \]  

(4)

A number of papers have simply assumed \( \tilde{q}_t \) to be constant (i.e. \( \tilde{q}_t = \alpha \)). This strand of research generally finds an absence of a cointegrating relationship for the vector implied by (4) when the Engle-Granger two-step method is used (see, inter alia, Meese and Rogoff (1988), Edison and Pauls (1993), Throop (1994) and Coughlin and Koedijk (1990)), but somewhat stronger evidence when the maximum likelihood estimator of Johansen is employed (see, inter alia, Edison and Melick (1992,1995), MacDonald (1997), Juselius and MacDonald (2000)). Studies which model \( \tilde{q}_t \) as a function of 'real' fundamentals such as net foreign assets and Balassa-Samuelson effects generally find more favourable long-run cointegrating relationships (see Chinn and Johnston (1999) and Clark and MacDonald (1999)).

The permanence of real exchange rates has been addressed in a somewhat separate strand of the empirical exchange rate literature. In particular, a number of researchers have used both univariate and multivariate Beveridge-Nelson decompositions to decompose real exchange rates into permanent and transitory components (see, for example, Huizinga (1987), Cumby and Huizinga (1990), Clarida and Gali (1994) and Baxter (1994)):

\[ q_t = q_t^P + q_t^T, \]  

(5)

where \( q_t^P \) and \( q_t^T \) are the permanent and transitory components of the real exchange rate. The general tenor of these results is that when a univariate decomposition is used the permanent component of the real exchange rate is around 0.9 (see, for example, Huizinga (1987)), and when a multivariate decomposition is used the split between the permanent and transitory components is more evenly balanced (see, for example, Clarida and Gali (1994)). These results would therefore seem to reinforce the unit root results: real exchange rates are highly persistent although they do contain important mean-reverting, or transitory, elements. A somewhat different approach to decomposing the permanent
and temporary components of the real exchange rate has been advocated by Clarida and Gali (1994). In particular, they consider the vector:

$$\Delta x_t' = [\Delta y_t, \Delta q_t, \pi_t],$$

where $y_t$, denotes relative output (home-foreign) and $\pi_t$ denotes relative inflation. Using a trivariate VAR modeling approach and the identification methods of Blanchard and Quah on the long-run matrix $C(1)$, Clarida and Gali are able to identify three shocks from this vector: a supply shock, a demand shock and a nominal shock. The particular identifying restrictions used (based on a modified version of the Mundell-Fleming-Dornbusch (MFD) model) are: money, or nominal, shocks do not influence the real exchange rate or relative output in the long run; only supply shocks are expected to influence relative output levels in the long run; both supply and demand shocks are expected to influence the real exchange rate in the long-run. Using this framework Clarida and Gali seek to explain the real US dollar bilateral rates of the Canadian dollar, German mark, Japanese yen and UK pound for the recent floating period.

Using a variance decomposition of the variables(6), and interpreting the sum of the demand and money shocks as the business cycle related components of the variance of the real exchange rate, then Clarida and Gali’s decompositions indicate that for all four real exchange rates the business cycle component constitutes approximately 90 per cent of the variance of the exchange rates after 40 quarters. Of this total, almost all is attributable to demand shocks in the case of the UK and Canada, while for Japan the split is 60% demand and 30 % monetary, while the split being approximately equal for the German mark. The proportion of the forecast error variance due to the supply shock is statistically insignificant at all forecast horizons. The very small supply side specific component reported by Clarida and Gali has been confirmed by others (see, for example, Chadha and Prasad (1997) and MacDonald and Swagel (1998)) for different to currencies and different time periods and has indeed become something of stylized fact in the literature on the economics of real exchange rates. However, one reason why CG find such a small supply side component may reflect the actual specification of the supply side used in their model. Both Rogers (1995) and Weber (1998) have reworked the CG analysis using a richer supply side specification and find that the supply side, or permanent component of the real exchange rate puts in a much more respectable showing of approximately 30 per cent.

We summarize this section by noting that, on the basis of a number of different approaches, real exchange rates are highly persistent. Vari-
ous interpretations have been proposed to explain this persistence, such as recognizing non-linearities in real exchange rates, the importance of pricing to market policies of companies and the relationship between the persistence in real fundamentals and real exchange rates. In this paper we pursue the latter interpretation in the context of a structured VAR approach. In contrast to previous estimates of such VARs we make, at least, two contributions. First, we explicitly condition the real exchange rate on a real interest differential, a variable which has been ignored in previous structured VARs and, secondly, we explicitly recognize the potential long-run or cointegrating relationships amongst the variables entering the VAR.

3 A simple model

In order to motivate our empirical analysis we present in this section a model usually referred to as the Mundell-Fleming-Dornbusch model (the particular representation is due to Clarida and Gali (1994) and Lee and Chinn (1998)). Essentially this model augments a standard textbook Mundell-Fleming model with sluggish price adjustment and forward looking expectations. The model is given by the following relations:

\[ y_t^d = \eta(s_t - p_t) - \sigma r_t, \]  
\[ p_t = (1 - \theta)E_{t-1}p_t + \theta p_t, \]  
\[ m_t - p_t = y_t - \lambda i_t, \]  
\[ i_t = E_t \Delta s_{t+1} + u_t, \]  
\[ r_t = (i_t - E_t (p_{t+1}) - p_t). \]

Equation (7) is an IS-relation, relating aggregate demand \((y^d_t)\) to the real exchange rate \((q_t \equiv s_t - p_t)\) and the expected real interest rate \(r_t\). Equation (8) is just the price adjustment equation where the bar denotes the permanent component of the price level \((p_t)\). A money market equilibrium condition is given by equation (9), while (10) is an uncovered interest parity condition augmented by a catch-all variable \(u_t\) that captures deviations from uncovered interest parity. Below, we will elaborate on the role of \(u\).

The supply side of the model is specified by two random walks:

\[ y_t^s = y_{t-1}^s + z_t, \]  
\[ 7 \]
\[ m_t = m_{t-1} + v_t, \]  
where \( z_t \) and \( v_t \) denote supply and money shocks, respectively.

The steady state of this model is given by:

\[ \mu = y_t', \]

\[ \gamma = \frac{1}{\eta} [\gamma + \sigma \tau_t], \]

\[ \tau_t = m_t - \mu - \lambda_t. \]

It is worthwhile contemplating the implications of the long-run solution of the above model for the time series properties of the variables of interest. If, as most empirical studies suggest, \( y_t \) is integrated of order one, I(1), then the real exchange rate and output, and possibly the real interest rate, should be cointegrated. Furthermore, the model implies that output, the price level, money and the nominal interest rate cointegrate.

Note further that in this model the long-run real interest rate, \( \tau_t \), equals \( \mu_t \). As long as \( u \) is a non-integrated process, the real interest rate should also be a stationary process. To the extent that we interpret all variables except \( s_t \) as measured against the rest of the world, the above model implies a stationary interest rate differential.

### 4 The empirical model

The above model suggests that there may be important interrelationships between real exchange rates, real interest differentials and real output differentials. In this section we consider tri-variate vector autoregressive models of output, interest rate differentials and the real exchange rate for the G7 countries.

Our general setup will be that of a cointegrated VAR, or vector error-correction model, of the form:

\[ \Gamma(L) \Delta X_t = \alpha \beta' X_{t-1} + \epsilon_t, \]

where \( X_t = [(y - y^*), q, (r - r^*)]'_t \), and the asterisk denotes the rest of the world (RoW). The other parameters of the model are \( \Gamma(L) \) which is a 3 × 3 matrix polynomial in the lag operator \( L \) and the matrix of cointegrating vectors \( \beta \) and the error-correction loading matrix \( \alpha \) as well as the i.i.d. disturbance vector \( \epsilon \), with covariance-matrix \( \Omega \).
4.1 The Data and their properties

We use quarterly data for the G7 countries, the United States, Japan, Germany, France, Italy, the United Kingdom and Canada, over the period 1978:Q2 to 1997:Q4. The real exchange rate series are real effective exchange rates from the IMF’s International Financial Statistics (line reu), the output data measure real GDP and are also from International Financial Statistics, denominated in domestic currency (code 99B).

The construction of an output differential vis-a-vis the rest of the world would require that we use some measure of real exchange rates to construct the RoW aggregate. This might induce some spurious comovement between the real exchange rate and the output differential that we want to avoid. We therefore constructed the real output differential as the cumulated growth differential vis-a-vis the United States. For the United States themselves, we looked at the cumulated growth differential vis-a-vis Japan and a weighted Japan-Germany average. The results were almost identical. The results for the United States reported in this paper are based on the differential vis-a-vis Japan.

The nominal interest rates are long bond yields (line 61) and the price indices are consumer prices (line 64). Foreign prices and foreign interest rates are in ‘effective’ units and have been constructed by aggregating the remaining G7 countries (i.e. exclusive of the home country) using the weights implicit in the effective exchange rates. Furthermore, we used the consumer price indices and long-term real interest rates to construct real interest rates: for each country we estimated a VAR in the process $Z'_t = \left[ \begin{array}{c} i_t, i^*_t, \Delta p_t, \Delta p^*_t \end{array} \right]$ and approximated $E(\Delta p_t)$ through forecasts from this VAR.

Figures 1-7 provide plots of the data for all seven countries. A first optical inspection reveals a rather close comovement between the output differential and the real exchange rate in most countries. In fact, we will see that this relationship is formally confirmed in our cointegration results. The comovement between the real exchange rate and real interest differentials also appears quite pronounced. For example, it is quite evident for the US for most of the sample and for parts of the sample for other countries.

In specifying the appropriate lag length of the VAR in $X_t$, we relied on standard information criteria. Since all of those suggested the use of either 2 or 3 lags for all countries, we decided to estimate the VAR with 2 lags throughout and to include a set of seasonal dummies. For Germany we also included a step dummy starting in 1990:Q1 to account for the effects of German reunification. In the VAR for Italy, we included a step dummy after 1992:6 to take account of the effect of the EMS-crisis.

Using a VAR specification with an unrestricted constant and without
trend, we then proceeded to implement Johansen’s test for cointegration. Table 1 contains the results. The data support the presence of at least one cointegrating relationship for five countries, with Japan and Canada being exceptions. We discuss our choice of the number of cointegrating relationships against the background of the theoretical model in the next sub-section.

4.2 The Structural Model

Our theoretical model predicts that only one common trend drives $X_t$ and that the cointegrating space is spanned by:

\[ \beta' = \begin{bmatrix} 1 - \eta \\ 0 \\ 0 \\ 1 \end{bmatrix}. \] (17a)

The second column of $\beta$ formalizes the trivial cointegrating relationship implied by the requirement that the interest rate differential is $I(0)$ as $u$ is assumed to be $I(0)$.

The tests reported in the previous section do, however, generally support the presence of one cointegrating relationship. To make these results consistent with the theory, we need to reconsider the interpretation of the catch-all variable $u_t$. Only if we allow $u_t$ to follow an $I(1)$ process can we rationalize $I(1)$-ness in real interest rates, as we find in the data. One interpretation of $u_t$ would be that of a risk-premium. However, this would be hard to reconcile with non-stationarity in the sample of countries and the time period we look at.

We suggest an alternative interpretation: recent research documents that the de-facto behaviour of central banks can be well approximated by a real interest rate rule (see e.g. Romer (2000) and Rotondi (2000) and the literature surveyed there for theoretical expositions and Clarida, Gali and Gertler (1998) for empirical evidence). If real interest rates are generated by such a rule, they may become very persistent and virtually indistinguishable from an $I(1)$ process.

Other factors may affect $u_t$. We could also think of it as capturing some notion of a speculative shock that drives a wedge between fundamentals and realizations of the spot exchange rate. This notion of misalignment has recently attracted renewed attention (see, for example, Clark and MacDonald (1999), Alberola et. al. (1999) and Weber (1997) ). Departures from uncovered interest parity are equally well documented (see e.g. Faust and Rogers (1999)) and are almost certainly going to be captured by $u_t$.

In this paper we are not very specific about the nature of $u_t$. We simply argue that models like that presented in Section 3 can potentially be reconciled with what appears - at least in typical macroeconomic
sample sizes - as non-stationary behaviour in real interest rates. In our empirical implementation we therefore follow the path along which the tests reported in the previous section direct us. We decided to impose one cointegrating relationship in the estimation of our model for all seven countries.

4.3 Model identification

Even though, as in Clarida and Gali (1994), we are dealing with a three-variable setup, the informational requirements for just identification are reduced due to the presence of cointegration; in fact, the presence of cointegration enables us to just-identify our model using a version of Blanchard-Quah’s (1989) procedure. To see this, note that the permanent component (extracted by means of the Beveridge-Nelson (1981) decomposition) of \( X_t \) can be written as:

\[
X_t^p = A_0 \sum_{l=0}^{t} \pi_l,
\]

where \( \pi_l \) is the vector of permanent shocks and \( A_0 \) is the long-run loading matrix. In our three-dimensional system with one cointegrating relationship, \( A_0 \) is \( 3 \times 2 \) and \( \pi_l \) is \( 2 \times 1 \). The permanent shocks are just a linear combination of the reduced-form residuals given by:

\[
\pi_l = \alpha'_\perp \varepsilon_t,
\]

and from Johansen (1995) the loadings matrix is given by:

\[
A_0 = \beta'_\perp (\alpha'_\perp \Gamma(1)\beta'_\perp)^{-1}.
\]

In the above \( \beta'_\perp \) and \( \alpha'_\perp \) are the orthogonal complements of \( \beta \) and \( \alpha \), respectively. These are, however, only determined up to a linear transformation. Hence, \( X_t^p \) remains the same whenever we choose \( \pi_l = S\pi_l \) and \( A_0 = A_0 S^{-1} \) for any non-singular \( 2 \times 2 \)-matrix \( S \). So, for any initial choice of \( \beta'_\perp \) and \( \alpha'_\perp \), how should we choose \( S \)?

We start by requiring that the two permanent shocks be orthogonal and have unit variance. Hence, we get

\[
\text{var}(\pi_t) = S\alpha'_\perp \Omega\alpha'_\perp S' = I_2.
\]

This gives us three non-redundant restrictions on \( S \). The fourth restriction that is required to just-identify the four elements of \( S \) comes from the theoretical model: in the presence of I(1) real interest rates, money shocks can have a “long-run” impact on the real exchange rate but not on the output differential. Requiring the first of the two permanent shocks
to be the supply shock and the second the money (nominal) shock and bearing in mind the ordering of variables in $X_t$, this amounts to requiring that:

$$\bar{A}_0 = \begin{bmatrix} a_{11} & 0 \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix} = A_0 S^{-1}. \quad (21)$$

This completes the identification of the permanent shocks. To just identify our model, we also need to identify a third shock which will be purely transitory. It arises naturally by requiring that it be orthogonal to the permanent shocks. Hence, the transitory shock is given by

$$\tau_t = \frac{\alpha' \Omega^{-1} \varepsilon_t}{\alpha' \Omega^{-1} \alpha} \quad (22)$$

where the denominator ensures that $\text{var}(\tau_t) = 1$.

5 Empirical Results

As mentioned earlier, we imposed one cointegrating relationship in the estimation of all seven models. In light of the discussion in section 4.2., it is of particular interest to check whether i) the real interest differential is I(0) and ii) or whether there is a genuine cointegrating relation between the real exchange rate and the real interest rate differential, i.e. $y$ can be excluded from the cointegrating relationship. The results of these tests are given in table 2.

It is interesting to note that for none of the countries can we actually reject the non-stationarity of the real interest rate differential, i.e. the joint exclusion restriction $\beta_1 = \beta_2 = 0$. This finding is in stark contrast to the literature, discussed in Section 2, which has, in general, failed to establish the link suggested by sticky price theories of exchange rate determination (in particular, Dornbusch’s (1976) overshooting model). However, the way in which the real interest rate enters the equilibrium relation is different across countries. For three countries - the US, France and the UK - we cannot reject the exclusion restriction on $y$, implying that the data support the presence of a genuine cointegrating relationship between $q$ and $r - r^*$. For the other countries in our cross-section the relationship between real interest rates and real exchange rates cannot be adequately captured without accounting for the real output differential.

Even though table 2 suggests that the data allow us to restrict the model further in individual cases, we actually left the estimated cointegrating vector unrestricted as we moved on to identify the model.

In figure 8 we provide the 'typical', i.e. cross-sectionally averaged dynamic response of the system for all G7 economies. The two permanent
shocks pass the 'duck test'\textsuperscript{2}: the real shock generally leads to a permanent increase in output relative to the rest of the world, coupled with a real appreciation and an increase in real return. This is in line with what one should see in response to a shock to total factor productivity. As for the nominal shock, the temporary expansion in output is coupled with an impact depreciation and a decrease of the real interest rate, and this is in line with what one would expect from a monetary disturbance.

For the temporary shock a structural interpretation is not very straightforward: this shock will temporarily widen the interest rate differential and lead to a depreciation coupled with a temporary increase in output. This could possibly be read as the response to a positive fiscal shock.

However, given the aggregated nature of our model, we do not interpret the identified shocks as technology, money and fiscal shocks. Rather, we think of them as amalgams and we refer to them as real, nominal and transitory shocks throughout the remainder of the paper. What is interesting, however, is that both real and nominal shocks can have potentially lasting effects on the exchange rate.

As we move on to the variance decompositions of $\Delta X_t$, contained in table 3, a few interesting points stand out: the second permanent shock - the nominal or monetary disturbance - explains quite a sizeable proportion of real exchange rate variance at all forecast horizons and for most of the seven countries. This finding is in line with Rogers (1999) who finds that monetary shocks - as one prime representative of nominal shocks - are an important source of real exchange rate variability. Economic theory emphasizes the role of monetary shocks for real exchange rate dynamics, but much of the earlier work, most notably Clarida and Gali (1994), could not empirically establish this important result. In US data, Rogers finds that monetary shocks explain between 20 and 60 percent of exchange rate variability and fiscal and productivity shocks combined account for between 5 and 25 percent. Our results confirm these findings for a cross-section of seven economies.

It is noteworthy how stable the share of variance that is explained by each permanent shock is over time. If there is any variation over time, it takes place in the first four quarters after which the variance shares of the various shocks reaches its permanent level.

5.1 Permanent Components and misalignment

The econometric approach taken in this paper, allows us to decompose the real exchange rate into permanent and transitory components without having to recur to non-testable restrictions. The recent literature on cointegrated systems shows that the transitory part of a multivariate

\textsuperscript{2}If it walks like a duck and quacks like a duck, it might actually be a duck.
time-series can be expressed as a linear combination of the deviation of the cointegrating relationships from their mean, i.e. the cointegration or equilibrium error (see Gonzalo and Granger (1995), Proietti (1997) and Johansen (1997)). In this paper, we use Johansen’s (1997) modification of the Gonzalo-Granger decomposition:

\[ X_t = C(1)\Gamma(1)X_t + [I - C(1)\Gamma(1)] X_t \]  

(23)

Proietti (1997) has shown that

\[ [I - C(1)\Gamma(1)] X_t = \psi \beta' X_t \]

which implies that - very much as in the Gonzalo-Granger decomposition - the transitory part of \( X_t \) is a linear combination of the cointegrating error.

In the context of this paper, the cointegration error, \( \beta' X_t \), measures the deviation of the data from the steady-state relationship (14). Hence, it seems an appealing idea to interpret \( \beta' X_t \) as a measure of exchange rate misalignment. Recent work by Alberola et al. (1999) has followed this approach. But a word of caution is in place here: it has become common practice in recent empirical research to treat the real exchange rate as an integrated variable. Interpreting the cointegration error as a measure of misalignment will then confine misalignment to purely transitory movements. This, however, runs against the very notion of a misalignment which is based on a persistent deviation from macroeconomic fundamentals. Also, the econometric permanent component we identify from an interdependent system, such as a cointegrated VAR, implicitly takes account of the fact that misalignments may feed back into economic fundamentals. As a result, the permanent component of the real exchange rate that gets identified from a multivariate decomposition may not represent the equilibrium rate that would prevail if certain shocks that are considered ‘non-fundamental’ by the econometrician had not occurred.

A method to measure the equilibrium exchange rate as the exchange rate that would have prevailed net of non-fundamental shocks and that implicitly takes account of potential feedback is advocated in Clarida and Gali (1994). These authors decompose the real exchange rate into structural components, i.e. that part of \( q \) that can be attributed to nominal and real shocks respectively.

In this paper we will employ the two approaches simultaneously. Comparing these concepts of misalignment will then enable us to say something about the extent of hysteresis that is induced by disturbances to the real exchange rate: if misalignments are small under the
cointegration-based measure, but large under the Clarida-Gali measure, then shocks that drive the exchange rate away from its fundamental value (in the Clarida-Gali concept) have an impact on the permanent value of fundamentals. Furthermore, we can then compare the persistence of the exchange rate misalignments that is implied by the various concepts.

Figures 9-15 give the results of the various real exchange rate decompositions for the G7. Panel a) plots the real exchange rate against the permanent component extracted using (23), panel b) the corresponding transitory component, i.e. a scalar multiple of $\beta'X_t$. Panel c) plots the real exchange rate against the historical component of $q$ that is due to the real shock. Hence, following Clarida and Gali, we are treating the permanent nominal and the transitory disturbances as 'non-fundamental'. This is in line with the interpretation of the catch-all disturbance $u_t$ that is likely to capture both demand side, monetary policy and financial market disturbances.

In table 4 a) and b) we give important descriptive statistics for the two measures of misalignment: The mean of their absolute values, their variance, autocorrelation and cross-correlation as well as the half-life implied by this autocorrelation.

The two misalignment measures are generally highly, although far from perfectly correlated. The CG measure is generally more persistent. The difference in half-life is particularly pronounced for Germany, France, Italy and Canada. Hence, the data suggest that non-fundamental economic shocks have a pronounced effect on fundamentals and are therefore likely to change the equilibrium exchange rate.

The results in table 4 would seem to go a considerable way towards a resolution of the real exchange rate puzzle; i.e. the perceived slow mean-reversion of real exchange rates referred to in Section 2. In our cross section of G7 economies the average half-life of $\beta'X_t$ is no more than 6 quarters, whereas the half-life of the CG-type transitory component is close to 10 quarters on average. Even the latter number is around half of that reported in empirical studies which extend the span of the data, either by taking a long historical run of data or by using panel methods.

By using a forecast error variance decomposition of changes in misalignment of $\Delta \beta'X_t$, we can shed some light on the sources of shocks driving the observed equilibrium error, $\beta'X_t$. This decomposition is presented in table 5. It would appear that transitory shocks account for the bulk of the equilibrium forecast error variance. In some cases, such as France and Canada, nominal shocks seem to play a role as well. Only in the UK does the real shock contribute in a meaningful way to equilibrium error variance. Very much as in the case of $\Delta q$, the variance shares of shocks are very stable across forecast horizons. The average con-
tribution of permanent shocks to the overall variance of misalignments mostly ranges from around forty to seventy percent. This suggests that in most countries permanent shocks are an important determinant of real exchange rate misalignment, but to the extent that permanent shocks matter for misalignment, it is nominal shocks that have the main role to play in explaining misalignment persistence.

6 Conclusion

In this paper we have examined the interaction of G7 real exchange rates with real output and interest rate differentials in the context of a structured VAR. Even though these two variables generally feature in standard macro models as key determinants of real exchange rates, the literature has so far not examined them jointly in one compact econometric framework. A further novel feature of our approach is that we exploit cointegration between real differentials and the real exchange rate to identify transitory and permanent components of the real exchange rate. In our analysis we generally find support for one cointegrating relationship between the output and real interest rate differentials and the real exchange rate. In some countries, this cointegrating relationship can be restricted to the real exchange rate and the real interest differential alone. We believe that this is an interesting finding since much of the earlier literature (see e.g. Baxter (1994)) could not establish this link in a bi-variate context.

Cointegration enables us to identify equilibrium exchange rates as the permanent component of real exchange rates that is consistent with dynamic equilibrium. Furthermore, the presence of cointegration also allows us to identify real, nominal and transitory disturbances with only minimal identifying restrictions, based on Blanchard’s and Quah’s procedure.

We compare deviations from dynamic equilibrium with Clarida and Gali’s (1994) notion of misalignment. We find the latter generally to be more persistent than the measure of misalignment that is based on the cointegrating error. This evidence suggests that persistent deviations of real exchange rates from their equilibrium value can have feedback effects on the underlying fundamentals, hence altering the equilibrium exchange rate itself.

Our results demonstrate that treating the real interest rate differential as an integrated variable can be a useful empirical strategy. Standard sticky price models will generally not be able to rationalize this non-stationarity but we have argued that slowly changing stances of monetary policy and financial market disturbances can make the real interest rate observationally equivalent to an integrated process in macroeco-
nomic sample sizes. In this paper, we have turned this apparent problem into a virtue by exploiting it for the identification of a compact econometric system.

Summing up, it seems that real differentials provide a parsimonious representation of fundamentals for real exchange rates. Obviously, being parsimonious forbids us to assign a very specific structural interpretation to the various shocks we identify. In particular, one way our work could be extended in the future would be to explicitly recognize the separate roles of monetary and fiscal policy shocks.
References


from a Panel Data Set”, *Journal of Money Credit and Banking*, 28, 54-63.
Table 1: Tests for Cointegration

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h denotes the number of cointegrating relationships

Table 2

Tests of exclusion restrictions on $\beta' = [\beta_1, \beta_2, \beta_3]$

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Values reported are p-values. Accepted restrictions in bold.
### Table 3

Variance decomposition of $\Delta q_t$

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### Table 4:

Misalignments in the G7 - descriptive statistics

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<th>half life (quarters)</th>
<th>$\rho(\beta'X, \text{ma})$</th>
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Table 5
Variance decomposition of $\Delta \beta' X_t$

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Figure 1: US Data
Figure 2: The Japanese Data

Figure 3: The German Data
Figure 4: The French Data

Figure 5: The Italian Data
Figure 6: The British Data

Figure 7: The Canadian Data
Figure 8: Cross-sectionally averaged impulse responses for the G7 economies.

Figure 9: Decomposition of log effective real exchange rate - United States
Figure 10: Decomposition of log effective real exchange rate - Japan

Figure 11: Decomposition of log effective real exchange rate - Germany
Figure 12: Decomposition of log effective real exchange rate - France

Figure 13: Decomposition of log effective real exchange rate - Italy
Figure 14: Decomposition of log effective real exchange rate - United Kingdom

Figure 15: Decomposition of log effective real exchange rate - Canada