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A Call on Art Investments*

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Abstract:
The art market has seen boom and bust during the last years and, despite the downturn, has received more attention from investors given the low interest environment following the financial crisis. However, participation has been reserved for a few investors and the hedging of exposures remains dificult. This paper proposes to overcome these problems by introducing a call option on an art index, derived from one of the most comprehensive data sets of art market transactions. The option allows investors to optimize their exposure to art. For pricing purposes, non-tradability of the art index is acknowledged and option prices are derived in an equilibrium setting as well as by replication arguments. In the former, option prices depend on the attractiveness of gaining exposure to a previously non-traded risk. This setting further overcomes the problem of art market exposures being dificult to hedge. Results in the replication case are primarily driven by the ability to reduce residual hedging risk. Even if this is not entirely possible, the replication approach serves as pricing benchmark for investors who are significantly exposed to art and try to hedge their art exposure by selling a derivative.

JEL Classification: G11, G13, Z11

Keywords: Art Market, Art Index, Alternative Investments, Option Pricing

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1. Introduction

Art prices remain as intriguing as ever, but during the recent financial crisis the number of market participants has decreased and those who remain have halted their hunt for record prices. Nonetheless, given the recent drop in art prices and the generally low interest rate environment, the art market remains an attractive venue for exceptional returns. Apart from direct investments, the obstacles to investing into art remain high and hedging exposure to art market risk is nearly impossible. The paper proposes to overcome these hurdles by introducing an index-based derivative. Prices are derived in an equilibrium approach, which sheds light on the attractiveness of art exposure, as well as in a replication setting, which helps investors exposed to art gauge the price for hedging their art portfolio. As such, it is the first analysis to formally consider art index derivatives and apply the two considered pricing models to an art index that has been constructed based on one of the most comprehensive national art price data sets.

During the boom, especially the contemporary market was regularly in the news with new record sales having been recorded almost daily. Damien Hirst's auction of works sold directly from the studio or Francis Bacon's "Tryptichon" sold for a staggering USD 86.2 million is only one of many such examples. Since then, the number of buyers has melted away and by October 2009 the market was down about 40% compared to the previous year according to artprice.com. In fear of not finding a buyer, sellers are reluctant to offer high quality works for auction. However, when top quality is offered to the market, record sales are still possible. This is demonstrated by the sale of the Yves-Saint Laurent estate or the "View of the Outskirts at the Sea near Marseille" by Max Beckmann, which reached a new record for Beckmann landscapes at an auction in the fall of 2009.

With the market drop, art funds, such as *The Fine Art Fund*, reported substantial losses due to the inability to hedge their inventory. Similarly, auction houses' bought-in artworks are subject to inventory risk. By hedging their market dependence, both players would have benefited from a short position in the proposed call option on the art index. At the same time, the existence and emergence of new art funds shows a general interest for investing into art. Art is an alternative asset class that offers exceptional return potential and can be interesting from a diversification perspective. While an individual collector may buy a few art pieces, an art fund is interested in the exposure to a broad range of works, which is currently difficult to realize. The derivative would overcome this problem, and an investment strategy, in turn, could be implemented by taking a long position in the art index call.

From a pricing perspective, continuous time modeling can be used to derive indicative prices for agents interested in trading the proposed call. Consider an economy with an aggregate stock market and the non-tradable art index. A closed form equilibrium option value is derived based on the agent's preferences for payoffs in states of the world where the economy performs poorly. As such, the equilibrium pricing approach does not rely on notoriously difficult to hedge art market exposure. This is in contrast to option prices obtained from a portfolio replication strategy in an incomplete market, which are provided for comparison. The optimal hedging asset analysis, in this case, is complemented by correlations estimates between art index and S&P 500 constituents. Once subjected to the extensive and unique art index data, these techniques allow the pricing of the new product.

The remainder of the paper is organized as follows: Section 2 introduces the art market and the index which is chosen as underlying for the call option. The introduction of the call is motivated in Section 3 which also presents the two alternative pricing models and shows the main results of the paper. Section 4 describes a numerical application of the derived results. Concluding remarks are offered in Section 5. The appendix comprises derivations of the main results.

2. The Art Market Index

2.1. The Art Market

Before proceeding to the modeling and estimation of art price dynamics, it is worthwhile looking at some of the peculiarities of the art market. Overall, the art market size has been estimated to be over USD 3 trillion with an annual turnover of more than USD 50 billion (McAndrew, 2008). The major players in the market are auction houses, dealers, galleries, museums, and private collectors. Also financial institutions have recognized the importance of the art market for their private clients demonstrated by the fact that UBS, Deutsche Bank, Credit Suisse, and others all have set up art advisory services to accommodate high net-worth individuals' (HNI) demands. Distinguishing characteristics of the art market are market inefficiency, low liquidity, significant transaction costs[1], and high barriers to entry.

To reconcile the inherent inefficiency of the art market with the standard treatment of the issue, recall that efficient markets in the sense of Fama (1970) imply that all available information is incorporated into the price and that price movements are random; today's price is the best predictor of tomorrow's price. For US stock returns Goyal and Welch (2002) and Timmermann and Granger (2004) offer support for non-forecastability. Cochrane (2006), on the other hand, finds evidence for predictability of asset returns. Especially long-run returns seem to be forecastable for stocks, in accordance with Campbell and Shiller (2001) and Claus and Thomas (2001).

For art returns the opposite seems to be true. Chanel, Gerard-Varet, and Ginsburgh (1994) suggest that, while stock returns are driven by fundamentals whose predictability increases with horizon, the art market is about tastes which may well be predictable for short horizons but probably not for future generations. As taste is subjective, its incorporation in the information set of the buyer seems hard, if not impossible. Therefore, prices that are governed by tastes, or preferences for characteristics in general, do not adhere to the EMH (Daniel and Titman, 2000). This might provide a rationale for increased activity in the art market beyond mere purchases for aesthetic returns.

[1]The commission charged by, e.g. Sotheby's is 25% on the first USD 20,000, 20% of the next USD 20,000 to USD 50,000, and 12% on the rest.

No matter what drives the direct demand for art, let it be investment opportunities or non-financial motivations, especially profit-driven buyers of art will want to gain or hedge market exposure and participate in its development. Given the market's low liquidity, only a derivative on a standardized underlying offers this desired broad exposure without incurring massive transactions costs. Examples from recently introduced mortality derivatives or weather derivatives show that the underlying need not be traded at all for the instrument to be a success. Finally, it is worthwhile noting that there have already been first attempts to introduce art futures based on the Mei Moses All Art Index. The instruments traded on Intrade, a so-called prediction platform intended for betting on the outcome of various events. Unlike the derivative proposed here, no model backed the price discovery, though. Furthermore, Intrade serves more the betting than the financial community that is interested in hedging and gaining exposures.

2.2. Art Index Construction

Baumol (1986) argues that it is not possible to compute the *true* value of art, since art simply does not pay a dividend that can be discounted. Nonetheless, in order to analyze art prices within the context of asset pricing theory, information is needed on the distribution of the asset returns. In order to allow for a comparison between returns on the art market and returns of stock and bond markets, numerous art price indices have been constructed. The most straightforward way to measure a price change is to calculate an average or median sales price of a sample of artworks in at least two subsequent periods. However, when the quality of the artworks included in the sample changes from period to period, severe problems arise. First, if for some reason, a disproportionate number of high-priced paintings have been sold in a given period, the median painting price would rise even if none of the painting's prices changed at all. Moreover, variation in the quality of artworks sold from period to period will cause the index to vary more widely than the value of any given artwork. Second, if there is a progressive change in the quality of artworks sold at different times, the index would be biased over time. Consequently, two basic approaches have been used in order to correct for the problem of changing quality, namely repeat sales regressions and hedonic price indices.

The repeat sales method measures the sales price difference of the same artwork between two periods (see e.g. Mei and Moses (2002)). This implies that the difference between transaction prices at two dates is a function solely of the intervening time period. The econometric model is an OLS regression of the natural logarithm of the ratio of the second sale price to the first sale price on a set of time dummy variables. The advantage of the repeat sales model is that it does not require the measurement of quality; it only requires that the quality of the individual assets in the sample is constant over time. However, artworks are generally held for long periods of time before they are resold. Consequently, a large part of the data has to be discarded because only one sale is recorded. In addition, data are lost because it is not always possible to match two or more transactions of the same artwork. This introduces a sample selection bias since relatively frequently transacting assets, such as Old Masters, are not representative of the larger population of the art market.

The hedonic approach implies that the quality of an artwork can be regarded as a composite of a number of different attributes. This means that artworks are valued for the utility that these characteristics bear. Hedonic prices are defined as the implicit prices of different attributes. The value of an artwork is the sum of the implicit prices for the different characteristics it possesses. Generally, one would assume these qualities to remain constant and that changes in how the market values these different characteristics is what makes the price of an artwork change The value of an artwork is the sum of the implicit prices for the different characteristics it possesses. The most important advantage of hedonic regression models, such as applied in Renneboog and Van Houtte (2002) and Hodgson and Vorkink (2004), is that they avoid the problem of selecting items of the same quality for comparison at different times. Furthermore, they do not discard data of assets that only have one recorded price, resulting in a larger sample size available for research. However, neither the set of hedonic variables nor the functional form of the relationship is known with certainty. This problem can result in inconsistent estimates of the implicit prices of the attributes with dramatic impact on the prediction of the value of artworks based on the hedonic price index.

Most debates on constructing art market price indices consider its methodological characteristics; not much has been said in the literature on the theory behind the constituents of the index. Ginsburgh and Moses (2006) argue that an art market index should outline general market trends, much like the Dow Jones Industrial Average describing the general direction of the US stock market. Such an optimal art market index would suggest an objectively defined criterion that poses minimal constraints on the selection of data. Besides representativeness, other important attributes of an index are liquidity and capacity. However, previous hedonic regression models include just the works of the most important or historically relevant artists in their hedonic art index. But why would an investor only be interested in works of the top 100 artists that have been found relevant by art historians? A better criterion from an investor's point of view would be the availability of the artworks, since then the index would represent those artists, which actually get traded in the market. Such an index would favor an artist selection criterion that is based on the number of trades, instead of the historic relevance.

Moreover, the traditional hedonic method of specifying artist dummies puts a constraint on the number of artists that can be included in the sample. For this reason, Kräussl and van Elsland (2008) have developed an alternative method to proxy for artistic value. Just as the average price of art per year is corrected for quality using the hedonic method, the average price of art per artist is corrected for quality in the same way. Their approach consists of 2 steps. As a first step, a new artistic value variable is created by adjusting the average price per artist for quality. The second step is to replace the artist dummy variables by the new continuous artistic value variable and to estimate an index that utilizes the entire sample, which leads to a better representation of the total art market. The 2-step hedonic approach by Kräussl and van Elsland (2008) enables the researcher to use every single auction record, instead of only those auction records that belong to a sub-sample of selected artists. This results in a substantially larger sample available for research and it lowers the selection bias that is inherent in the traditional hedonic and repeat sales methodologies.

The index created from the 2-step approach is, thus, possibly one of the most comprehensive performance indicators of the art market, and it is, therefore, used as underlying for the proposed derivatives structure. Before the pricing is explored in more detail, the data used for the index construction is described in the following.

2.3. Data

To construct the 2-step hedonic index auction records from `www.artnet.com` for all artists with German nationality have been used.[2] For each auction record, the following characteristics were available: artist name, artist nationality, artist year of birth, artist year of death (if applicable), title of work, year of creation of the work, support, technique, dimension 1 (either height or width), dimension 2 (either height or width), miscellaneous (containing info on whether the work is signed, stamped, etc.), auction house, date of auction, lot number, low prior estimate of auction price, high prior estimate of auction price, sale price, currency of sale price, sale price converted to dollars and a note on the sale indicating whether it was bought in, withdrawn, sold at hammer price or at a premium.

The initial number of downloaded auction records over the years 1985 to 2007 was 120,688, covering data of 541 auction houses and 7,849 German artists. Of these records, 43.5% were either works that have been bought-in or withdrawn. For another 1.4% of the auction records, no sales price was communicated. This reduces the number of available records to 66,471. Further, 5,296 records were deleted due to missing data on either one of the hedonic variables used in the analysis. This results in a complete sample of 61,135 auction records of 5,115 different German artists. To the best of our knowledge, the largest sample that has been used in previous work to estimate a national hedonic art price index consists of 37,605 observations and is used in Higgs and Worthington (2005).

The dependent variable used in all hedonic models is the natural logarithm of the sales price converted to USD. The hedonic variables describe the following characteristics: surface, type of work, artists' reputation, attribution, living status, and auction house. The resulting semi-annual *German Art All* index, which forms the basis for the call option, has been estimated by OLS using robust standard errors. It is depicted in Figure 1.

Figure 1 about here

[2] The focus on artists of German nationality is motivated by data availability, but without loss of generality in terms of applying a derivative structure to an art index.

3. Pricing Model for the Art Index Derivative[3]

With derivatives being in zero net supply, successful introduction requires both, long and short, positions to be interesting to investors. The buyer of the option pays the option price, which will be determined below, and is then entitled to receive the cash amount by which the art index exceeds the strike price at maturity. The seller, on the contrary, needs to provide this difference if the option finishes in the money.

On the demand side, investors who are interested in gaining exposure to the general art market are possible candidates. It lets them trade a previously non-available risk factor, which can be interesting from a diversification perspective. For sufficiently low correlation, this can even be interesting for institutional investors. Furthermore, the derivative opens the possibility to participate in a market that may offer significant return potential after the recent drop. Lastly, art funds may want to take a long position when betting on the relative underperformance of an artist.

Investors willing to take a short position (i.e. write the call) should be those who wish to hedge their art exposure in a downturn. This is interesting for everyone with significant art inventory. Here not only art funds would find some protection from falling prices useful, but also auction houses that own a large number of bought-in pieces or banks engaging in art-financing can benefit from a short position.

Matching long and short positions, demand and supply will eventually determine the market price of the contract. Beforehand, however, market participants need to form a belief about what the call is worth. Therefore, a formal model is needed to capture the value of the option. In a standard option pricing setup, the writer can hedge his exposure by taking an appropriate position in the underlying. The replication strategy of Merton (1977), however, rests on the assumption that one is able to hold the art index in a similar fashion as one can hold a portfolio of stocks replicating some equity index. Due to the nature of the art market, it is not possible to buy art works that mirror the art index performance; pieces are unique, transaction cost tremendous, and traders are non-marginal. Consequently, unless the seller already owns a significant art portfolio, one can, at best, hope for finding a correlated asset that is traded and that can be used for hedging purposes. In presence of a sufficiently correlated asset, this pricing strategy prescribes easily implementable positions to hedge the derivative, such that writing the call would not only be economically meaningful for those with significant art market exposure. Since one cannot generally expect such an asset to exist, one can also derive option prices from the stochastic discount factor in an equilibrium setting. This has the further advantage that one can analyze the value of being exposed to a previously non-traded risk. Both approaches are considered subsequently for comparison, beginning with the equilibrium setup.

[3]Note that the pricing results of this section do not depend on a particular choice of index. In fact, they are applicable for a wider set of options on non- or partially traded underlyings such as real estate.

3.1. Art Option Pricing in an Equilibrium Setting

Consider an economy with a representative constant relative risk aversion (CRRA) agent who can invest into the money market account evolving at rate r and the stock market driven by a geometric Brownian motion[4] $dS_t = aS_t dt + bS_t dB_{(1,t)}$ with drift a, and variance b^2. Furthermore, there exists a traded call option $C(A, S, t)$ on the non-traded art index A_t. The evolution of the latter is given by

$$dA_t = \mu A_t dt + \sigma A_t dB_{(2,t)}. \tag{1}$$

B_1 and B_2 are two standard Brownian motions with correlation ρ. The dynamics of the call price follow from an application of Itô's lemma

$$\begin{aligned} dC &= \left(C_t + C_A\mu A + \frac{1}{2}C_{AA}\sigma^2 A^2 + C_{AS}\sigma bAS\rho + \frac{1}{2}C_{SS}b^2S^2 + C_S aS\right)dt \\ &\quad + C_A\sigma A dB_{(2,t)} + C_S bS dB_{(1,t)}, \end{aligned} \tag{2}$$

where subscripts of C denote partial derivatives. For notational ease, let the drift of Equation (2) be μ_C.

In this standard setup, the investor faces the problem of maximizing expected lifetime utility (EU) by choosing the fraction invested into the stock index $\pi_{(1,t)}$, the call option $\pi_{(2,t)}$ as well as consumption c_t

$$\sup_{\{c_t, \pi_{(1,t)}, \pi_{(2,t)}\}} E\left[\int_0^T U(c_t)dt\right] \tag{3}$$

subject to his wealth dynamics dW_t

$$dW_t = \left[W_t(r + \pi_{(1,t)}(a - r) + \pi_{(2,t)}(\mu_c - r)) - c_t\right]dt + \pi_{(1,t)}bW_t dB_{(1,t)} + \pi_{(2,t)}\sigma C_A W_t dB_{(2,t)}.$$

The solution to the portfolio problem is given in Appendix A.

In the equilibrium, the agent holds the stock and derivatives must be in zero net supply, such that $\pi^*_{(1,t)} = 1$ and $\pi^*_{(2,t)} = 0$. From the equilibrium, the pricing kernel m_t of the economy can be obtained[5] and is of the following form

$$dm_t = -rm_t dt - \gamma bm_t dB_{(1,t)}. \tag{4}$$

The continuous time pricing equation $E[d\,(mC)] = 0$, where C is the call price, gives

$$Cr = C_t + (\mu - \gamma b\sigma\rho)C_A A + (a - \gamma b^2)C_S S + \frac{1}{2}C_{AA}\sigma^2 A^2 + C_{AS}\sigma bAS\rho + \frac{1}{2}C_{SS}b^2 S^2. \tag{5}$$

[4]The choice of a geometric Brownian motion is convenient for analytical tractability, but could be relaxed if needed.
[5]See Breeden (1979) for details.

Substituting the equity premium relation (see Appendix A for the derivation of the equity premium) for the coefficient multiplying $C_S S$, one obtains the partial differential equation (PDE) that the call has to satisfy. The following proposition summarizes the result.

Proposition 1. ***Equilibrium Pricing Equation.*** *Under the assumption of a production economy with CRRA investor where the art index is not tradable, any derivative on the art index must satisfy the following PDE*

$$Cr = C_t + (\mu - \gamma b\sigma\rho)C_A A + rC_S S + \frac{1}{2}C_{AA}\sigma^2 A^2 + C_{AS}\sigma bAS\rho + \frac{1}{2}C_{SS}b^2 S^2, \tag{6}$$

given that one assumes the existence of a traded asset S that is correlated with the underlying.

Given the terminal condition $C(A, S, T) = (A - K)^+$, application of the Feyman-Kac theorem allows to express the solution to Equation (6) as

$$C(A, S, t) = \widetilde{E}\left[e^{-r(T-t)}C(A, S, T)|\mathcal{F}(t)\right]. \tag{7}$$

The following proposition shows that Equation (7) can be evaluated in closed form.

Proposition 2. ***Call Price in Equilibrium.*** *In a production economy with a representative CRRA agent, the closed form solution for a call option on the art index with strike price K corresponding to the fundamental PDE of proposition 1 is given by*

$$C(A, S, 0) = A(0)e^{-(r-\delta)}\Phi(d_1) - Ke^{-rT}\Phi(d_2), \tag{8}$$

where $\delta = \mu - \gamma b\sigma\rho$ and $d_{1/2} = \frac{\ln\frac{A_0}{K} + \left(\delta \pm \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$ and $\Phi(\cdot)$ is the cdf of a standard normal random variable.

A proof of the proposition is contained in Appendix B.

3.2. Art Option Pricing Using a Minimal Variance Approach

Instead of assuming a representative CRRA agent for option pricing purposes, one can rely on hedging arguments in an incomplete market. This may be especially interesting for investors writing the call to hedge existing art exposures. For instance, Cochrane and Saa-Requejo (2000) provide good-deal bounds for asset prices in incomplete markets. Bayraktar and Young (2008) investigate the pricing of options on non-traded assets, which closely matches the setup here. Applying these ideas, let the option value C again be a function of the underlying index A and the traded (and with A correlated) stock index S, whose dynamics are the ones given above. From here, an application of the "self-financing in the mean" argument allows for the derivation of a partial differential equation (PDE) that the option price has to satisfy.

Consider a portfolio of the stock and the money market account, approximating the option value; i.e. $C \approx nS + mB$. Self-financing in the mean implies that the gains and losses from the portfolio V should be approximately equal to the change in the option value

$$dC \approx ndS + mdB = dV, \tag{9}$$

where n and m are shares invested into the stock and the money market account, respectively. Substituting the call dynamics of Equation (2) into Equation (9) and replacing dS and dB, one wishes to choose n to minimize the variance of

$$\begin{aligned}&\left(C_t + C_A\mu A + \frac{1}{2}C_{AA}\sigma^2A^2 + C_SaS + \frac{1}{2}C_{SS}b^2S^2 + C_{AS}\sigma bAS\rho - Cr\right)dt\\ &\quad\approx nS(a-r)dt + (n-C_S)bSdZ - C_A\sigma AdW,\end{aligned} \tag{10}$$

where m has been replaced by $(C - nS)/B$. Since only the right hand side of Equation (10) contains random terms, minimizing the variance boils down to choosing n according to

$$n^* = \arg\min_n \operatorname{Var}\left[nS(a-r)dt + (n-C_S)bSdZ - C_A\sigma AdW\right]. \tag{11}$$

With

$$\operatorname{Var} = (n-C_S)^2S^2b^2dt + C_A^2\sigma^2A^2dt - 2(n-C_S)SbC_A\sigma A\rho dt \tag{12}$$

the optimal hedging share is given by

$$n^* = \frac{C_A\sigma A\rho}{Sb} + C_S. \tag{13}$$

Equation (13) implies the following variance for the hedging portfolio

$$\operatorname{Var}^* = C_A^2A^2\sigma^2(1-\rho^2)dt, \tag{14}$$

which is intuitively zero if and only if $|\rho| = 1$, implying that the underlying can be perfectly hedged. Since the variance of the hedging portfolio is generally non-zero, the writer of the call will be unable to completely hedge his risk.[6] Consequently, he should be reimbursed. Following Bayraktar and Young (2008), the compensation is chosen to be a constant multiple ϑ of the local standard deviation of the portfolio; ϑ can be interpreted as a risk aversion-coefficient. This setup implies that the drift of $dV - dC$ is equal to $(V-C)rdt + \vartheta\sqrt{C_A^2A^2\sigma^2(1-\rho^2)dt}$; i.e. the risk-free rate plus the additional risk compensation.[7] Together with Equation (11), one obtains the PDE that the option has to satisfy. The result is summarized in the following proposition.

[6] Even someone with significant art exposure will face the problem that the sensitivities of the option and his art portfolio to art price changes will not be identical.

[7] For deriving this result, note that the change in portfolio value is written as $dV = ndS + (C - nS)rdt - Crdt + Vrdt$.

Proposition 3. ***Fundamental Pricing Equation.*** *(Adapted from Bayraktar and Young (2008)) Since the art index is not tradable, it follows that the market is incomplete. Therefore, any derivative on the art index must satisfy the following non-linear PDE*

$$
\begin{aligned}
&C_t + C_A A\left(\mu - \sigma\rho b^{-1}(a-r)\right) + \frac{1}{2}C_{AA}\sigma^2 A^2 + C_S S r + \frac{1}{2}C_{SS}b^2 S^2 + C_{AS}\sigma b A S \rho \\
&= Cr - \vartheta \left|C_A\right| A\sigma\sqrt{1-\rho^2},
\end{aligned} \tag{15}
$$

given that one assumes the existence of a traded asset S that is correlated with the underlying.

The right hand side of Equation (15) shows that due to the incompleteness, the drift of the hedging portfolio is equal to something in excess of the risk-free rate.

Bayraktar and Young (2008) show that if the terminal condition is an increasing function of the underlying (e.g. as is the case for a call option), then $C_A \geq 0$ and the PDE becomes linear and of the form

$$
\begin{aligned}
&C_t + C_A A\left(\widetilde{\nu} + \vartheta\sigma\sqrt{1-\rho^2}\right) + \frac{1}{2}C_{AA}\sigma^2 A^2 + C_S S r + \frac{1}{2}C_{SS}b^2 S^2 + C_{AS}\sigma b A S \rho \\
&= Cr,
\end{aligned} \tag{16}
$$

with $\widetilde{\nu} = \mu - \sigma\rho b^{-1}(a-r)$. Equation (16) can again be solved in closed form by appealing to the Feyman-Kac theorem such that the solution with terminal condition $(A-K)^+$ is given by

$$
C(A,S,t) = \widehat{E}\left[e^{-r(T-t)}C(A,S,T)|\mathcal{F}(t)\right]. \tag{17}
$$

Analogously to the equilibrium case, Equation (17) can be evaluated in closed form. The solution gives the art index option value for the writer of the call.

Proposition 4. ***Call Price.*** *The closed form solution for a call option on the art index with strike price K corresponding to the fundamental PDE of proposition 3 is given by*

$$
C(A,S,0) = A(0)e^{-(r-\delta)}\Phi(d_1) - Ke^{-rT}\Phi(d_2), \tag{18}
$$

where $\delta = \widetilde{\nu} + \vartheta\sigma\sqrt{1-\rho^2}$ and $d_{1/2} = \frac{\ln\frac{A_0}{K} + \left(\delta \pm \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$ and $\Phi(\cdot)$ is the cdf of a standard normal random variable.

The proof follows along the same lines as the one of Proposition 2.

As the portfolio setup is presented from a writer's perspective, the price intuitively increases with risk multiple ϑ. It comes as no surprise that in an incomplete market, the price is not unique.

Lastly, if there is no reliable correlation estimate available, the agent can use the postulated art index dynamics of equation (1) to obtain a lower bound estimate for the call price given by

$$C(A,t) = E^{\mathbb{Q}}\left[e^{-r(T-t)}(A_T - K)^+|\mathcal{F}_t\right]. \tag{19}$$

The solution to equation (19) follows from the Black and Scholes (1973) setup and is given in the following proposition.

Proposition 5. ***Lower Bound Call Price.*** *Assuming the art index dynamics to satisfy equation (1) and a constant risk-free rate, the art index option price admits the well-known closed form solution*

$$C(0) = A_0\Phi(d_1) - e^{-rT}K\Phi(d_2), \tag{20}$$

where $d_{1/2} = \frac{\ln\frac{A_0}{K}+\left(r\pm\frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$ *and* $\Phi(\cdot)$ *is the cdf of a standard normal random variable.*

The price given by Proposition 5 constitutes a lower bound only, since the writer will not be able to hedge his position. It may serve as reference for the writer wishing to hedge his existing art portfolio. Due to the not perfectly matching sensitivities of art portfolio and derivative, he may, nonetheless, demand a premium.

In the following section the pricing implications of the derived results will be explored. The two model solutions are subjected to the art index data to study the impact of the correlation between the index and traded assets.

4. Numerical Application

4.1. Equilibrium Prices

In an economy with representative investor, equilibrium prices are given by Equation (8). A natural candidate for the production process of the economy is the S&P 500 index. Drift and diffusion parameters for the art index and stock dynamics along with a correlation estimate are given in Table 1.

Table 1 about here

Upon choosing a risk aversion parameter, option prices can be computed using the estimates in Table 1. Given the standard choice of $\gamma = 3$, option prices for different strike prices K and maturities are shown in Figure 2. These results are indicative for the prices at which a call option on the art index might trade.

Figure 2 about here

Additionally, by varying the risk aversion parameter it is possible to explore the effect of differences between investors' risk preferences on the price of the call option. For each value of the parameter the model provides an estimate of the risk premium demanded by investors. A comparison of the results for a risk neutral agent and his risk averse counterpart (Figure 3) shows that risk aversion reduces the equilibrium price of the call option.

Figure 3 about here

Even more interesting than the premiums in Figure 3 is the effect of the sign of the correlation between the art market and the general economy. To analyze this issue, the two extreme cases of $\rho = \pm 1$ are shown in Figure 4. Subplot (a), for a perfectly negative correlation, shows qualitatively similar numbers as in the reference case. The sign of the premium reverses when the correlation between the markets becomes positive. This is intuitive when recalling that premiums should depend on the marginal utility in good and bad states of the world. When correlation is positive, the call pays off when the economy, measured by the stock index, is in a good state and, thus, the agent only marginally increases his utility by the extra consumption possible. As is shown in Subplot (b), the investor requires a premium for holding the call. If the art market is negatively correlated with the stock market, the call pays off when most needed, and the agent is willing to accept a negative premium. In terms of prices, he is willing to pay a much higher price given that the call pays out when the general economy performs poorly. From a writer's perspective, the cash outflow in the bad state of the economy is only acceptable if the option price is sufficiently high.

Figure 4 about here

4.2. Minimal Variance Prices

If one is not willing to assume a representative investor, Section 3 has shown that option prices can also be computed given a correlated asset and an assumption about the risk multiple ϑ of the local standard deviation of the hedging portfolio.

Probably the most prominent candidate for such an asset that is expected to depend on the behavior of the art market is the Sotheby's stock.[8] Sotheby's, one of the most prominent auction houses of art in the Western world, is active in different segments of the art market. It is most likely best known for auctioning fine art, antiques, decorative art, jewelry and collectibles. However, it is also involved in the brokerage and financing of works of art

Sotheby's started trading on the 13th of May 1988 on the NYSE. Both volume and stock price have experienced a large increase since the start of 2003 when art markets started to boom. Figure 5 shows the price and volume developments. As is visible from the figure, there has been a decrease in both stock price and trading volume in recent years (2007-2008). Although art markets had not plummeted dramatically yet, as can be seen from the German Art All index (Figure 1), the change in both values could indicate the expectations of investors for 2009 to become a correction period.

[8]Other listed candidates include, for instance, Artnet. However, they lack sufficient historical data for parameter estimation.

Figure 5 about here

Comparing the Sotheby's figures with the art index development already hints at the stock price being affected by events that have hardly impacted the index development. Organizational aspects and Sotheby's performance in all of the previously mentioned segments, as well as macroeconomic shocks have an influence on the stock price, but not necessarily on the art price index. Even if the stock price decline mirrors investors' expectations about a shrinking art market, the visual inspection of the figures suggests that the art index behavior at most lags the stock price behavior, if there is a relation at all.

The visual inspection is confirmed when estimating the correlation of the art index and Sotheby's returns which is required for option pricing in the minimal variance approach. Using semi-annual returns of the traded asset that correspond to the updates of the index for the time period between 1988-2007, the estimated correlation coefficient is 5.5%. The estimate alongside with drift and diffusion parameters is shown in Table 1.

Whilst a low correlation is disappointing from a hedging perspective of the call option – a fact the significantly exposed writer may care less about – it also offers anecdotal evidence of the independence of the art market when compared to ordinary financial markets. This further motivates the attractiveness of the call on art investments from a diversification perspective.

In order to get a better impression of the effect of correlation, Figure 6 plots the dependence of price on degree of market incompleteness. The more the agent is able to reduce hedging portfolio risk, the lower the price of the call. This simple quadratic relationship stands in contrast to the equilibrium case, where prices are solely determined on the basis of payoffs in good and bad states of the economy.

Figure 6 about here

Despite the intuitive appeal of the Sotheby's stock as hedging candidate, the correlation estimate has turned out to be rather low, and it is worthwhile exploring whether there may be other, more suitable, stocks in the S&P 500 universe.[9] To investigate this issue, Figure 7 shows the distribution of correlation estimates between the art index and the constituents of the S&P 500 that were included in the index for a sufficiently long time during the sample period.

Figure 7 about here

First of all, Figure 7 confirms that the contemporaneous correlation between the art market and the stock market is very low. The vast majority of correlation estimates lies between -10% and 10%. The fact that the highest estimated similarity between the art and the stock market is found for a

[9]Although the art price index is a German index, the market for the included artists is a global one, such that the S&P is the more appropriate benchmark than the German DAX.

utility firm hints at the possibility of improving hedging performance by considering art unrelated industries.

The correlation analysis confirms that following a replication strategy maybe risky for the writer of the call. This is intuitive, given the low correlation; the writer of the call is exposed to a lot of residual risk if he cannot properly hedge his position with the traded asset. It is therefore also little surprising that the risk multiple ϑ exhibits a non-negligible impact on prices. Recall that call prices given by Proposition 4 require an assumption about ϑ. Unfortunately, theory, unlike in the equilibrium setting, offers little guidance for an appropriate choice. Therefore, Table 2 reports a selected number of option prices for different risk appetites. Prices are generally higher than in the equilibrium setting.

Table 2 about here

Although the minimal variance pricing approach prescribes a hedging strategy for the writer of the call, the low correlation makes prices very dependent on ϑ, as just mentioned. Only a writer who is already heavily exposed to the art market can hope for his exposure to compensate for the lack of traded assets for replication. As such, he may be willing to accept a rather small ϑ. The equilibrium approach, on the contrary, does not require a parameter that is hard to determine, and it yields prices based on the attractiveness of the art risk factor. These prices incorporate supply and demand pressures, and the higher the price the seller requires, the more the buyer is willing to spend without worrying about replicating the position. An additional advantage of the equilibrium approach is that it offers a possibility to quantify the benefit, in terms of the premium investors are willing to pay, of diversifying into the art market.

5. Concluding Remarks

This paper introduces and develops the pricing of a call option on one of the most comprehensive national art indices. The derivative allows investors to hedge their art market exposure by taking a short position in the instrument. On the demand side, who desire to diversify to diversify into art without having to incur tremendous transactions costs or betting on a single artist are likely to benefit from the new product.

From a modeling perspective, equilibrium pricing and hedging approaches are compared. In the former, prices are shown to primarily depend on the correlation between art and the economy on the basis of payoffs and marginal utilities in different states of the world. It is the desirability of exposure to the art risk factor that determines the economy's equilibrium price. The less art is correlated with the economy, the more compensation the option seller requires in order to give up the exposure. When seen as a function of risk aversion, differences in option prices have been shown to be an estimate of the art risk premium. In the second setup, a replicating portfolio is used to price the option. Despite the advantage of implying a hedging strategy for the derivative, the low correlation between art and economy makes prices particularly dependent on the level of a risk appetite coefficient for which theory offers little guidance. Only agents owning a substantial art collection may be willing to sell the call at, or close to, the derived lower bound. Furthermore, correlation only impacts prices through hedging performance, but does not take the attractiveness of the exposure gained through the derivative into account. An analysis of S&P 500 constituents shows that the art market and equity correlation is generally not exceeding $|10\%|$.

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Appendix A. Solution to the Portfolio Coice Problem

The HJB of the portfolio problem in (3) is given by

$$\begin{aligned} 0 &= \sup_{\{c_t,\pi_{(1,t)},\pi_{(2,t)}\}} \left\{ \frac{c_t^{1-\gamma}}{1-\gamma} - \beta V + \frac{\partial V}{\partial t} + \frac{\partial V}{\partial W} \left[W_t(r + \pi_{(1,t)}(a-r) + \pi_{(2,t)}(\mu_c - r)) - c_t \right] \right. \\ & \left. + \frac{1}{2}\frac{\partial^2 V}{\partial W^2} \left(\pi_{(1,t)}^2 b^2 W_t^2 + \pi_{(2,t)}^2 \sigma^2 C_A^2 W_t^2 + \rho \pi_{(1,t)} \pi_{(2,t)} b \sigma C_A W_t^2 \right) \right\}, \end{aligned} \quad \text{(A.1)}$$

where the value function is defined by $V(W,t) = \sup_{\{c(t),\pi(t)\}} E\left[\int_0^T U(c_t)dt\right]$ and β is the agent's determinant for time preference. The HJB implies the following first order conditions for optimal consumption c_t^* and portfolio choice $\pi_{(1,t)}^*$ and $\pi_{(2,t)}^*$, respectively

$$c_t^* = \left(\frac{\partial V}{\partial W}\right)^{-\frac{1}{\gamma}} \quad \text{(A.2)}$$

$$0 = \frac{\partial V}{\partial W} W_t(a-r) + \frac{\partial^2 V}{\partial W^2}\pi_{(1,t)}^* b^2 W_t^2 + \frac{1}{2}\frac{\partial^2 V}{\partial W^2}\pi_{(2,t)}\rho b \sigma C_A W_t^2 \quad \text{(A.3)}$$

$$0 = \frac{\partial V}{\partial W} W_t(\mu_c - r) + \frac{\partial^2 V}{\partial W^2}\pi_{(2,t)}^* \sigma^2 C_A^2 W_t^2 + \frac{1}{2}\frac{\partial^2 V}{\partial W^2}\pi_{(1,t)}\rho b \sigma C_A W_t^2. \quad \text{(A.4)}$$

The value function can be shown to be of the form $V(W,t) = g(t,T)^\gamma \frac{W^{1-\gamma}}{1-\gamma}$. Substituting this guess along with the optimal values c_t^*, $\pi_{(1,t)}^*$, and $\pi_{(2,t)}^*$ into (A.1), yields an ordinary differential equation for $g(t,T)$

$$\begin{aligned} 0 &= \gamma + \frac{\partial g}{\partial t}\gamma + g(t,T)\left((1-\gamma)\left(r + \pi_{(1,t)}^*(a-r) + \pi_{(2,t)}^*(\mu_c - r) \right. \right. \\ & \left. \left. - \frac{1}{2}\left((\pi_{(1,t)}^*)^2 b^2 + (\pi_{(2,t)}^*)^2 \sigma^2 C_A^2 + \rho \pi_{(1,t)}^* \pi_{(2,t)}^* b \sigma C_A \right) \right) - \beta \right) \end{aligned} \quad \text{(A.5)}$$

whose solution is given by $g(t,T) = -\frac{\gamma}{\chi} + e^{-\frac{\chi}{\gamma}\cdot(T-t)} \cdot \left(1 + \frac{\gamma}{\chi}\right)$ when $V(W,T) = \frac{W_T^{1-\gamma}}{1-\gamma}$ and χ represents the coefficient multiplying $g(t,T)$ in (A.5).

The equity premium is derived from Equation (A.3) by substituting the equilibrium conditions $\pi_{(1,t)}^* = 1$ and $\pi_{(2,t)}^* = 0$, yielding

$$a - r = \gamma b^2. \quad \text{(A.6)}$$

Appendix B. Proof of Proposition 2

For the considered call option with payoff $(A_T - K)^+ = A_T I_{\{A_T \geq K\}} - K I_{\{A_T \geq K\}}$, one can price the components of Equation (7) separately, implying

$$C(A,S,t) = \widetilde{E}_t\left[e^{-r(T-t)} A_T I_{\{A_T \geq K\}}\right] - K\widetilde{E}_t\left[e^{-r(T-t)} I_{\{A_T \geq K\}}\right] \quad \text{(B.1)}$$

where E_t denotes the conditional expectation operator and $I_{\{A_T \geq K\}}$ is the indicator function.

Consider first the second part of the right hand side of equation (B.1). Setting r to be constant[10], the discounting term can be taken out of the expectation and one is left with evaluating the probability of A_T exceeding K under the appropriate measure,

$$\begin{aligned} Ke^{-r(T-t)}\widetilde{E}_t\left[I_{\{A_T \geq K\}}\right] &= Ke^{-r(T-t)}\widetilde{\text{Prob}}\left(A_T \geq K\right) \\ &= Ke^{-r(T-t)}\widetilde{\text{Prob}}\left(\ln A_T \geq \ln K\right) \end{aligned} \tag{B.2}$$

To evaluate this probability, note that the risk neutral drift of A_t is given by the coefficient of C_A in Equation (6). A straight forward application of the Itô lemma gives

$$A_T = A_0 \exp\left\{\sigma \widetilde{W}_T + (\delta - \frac{1}{2}\sigma^2)T\right\} \tag{B.3}$$

from where the probability of A_T exceeding K at $t = 0$ follows according to

$$\widetilde{\text{Prob}}\left(\ln A_T \geq \ln K\right) = \Phi(d_2); \tag{B.4}$$

with $d_2 = \frac{\ln \frac{A_0}{K} + \left(\delta - \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$.

In order to evaluate the first part of Equation (B.1), one discounts by A_t and follows along the lines shown above to obtain

$$\begin{aligned} \widetilde{E}_t\left[e^{-rT}A_T I_{\{A_T \geq K\}}\right] &= A(0)e^{-(r-\delta)}\widehat{E}_t\left[I_{\{A_T \geq K\}}\right] \\ &= A(0)e^{-(r-\delta)}\widehat{\text{Prob}}\left(\ln A_T \geq \ln K\right) \\ &= A(0)e^{-(r-\delta)}\Phi(d_1) \end{aligned} \tag{B.5}$$

where $d_1 = \frac{\ln \frac{A_0}{K} + \left(\delta + \frac{1}{2}\sigma^2\right)T}{\sigma\sqrt{T}}$.

[10]This is without much loss of generality; compare analysis in Liu, Pan, and Wang (2005).

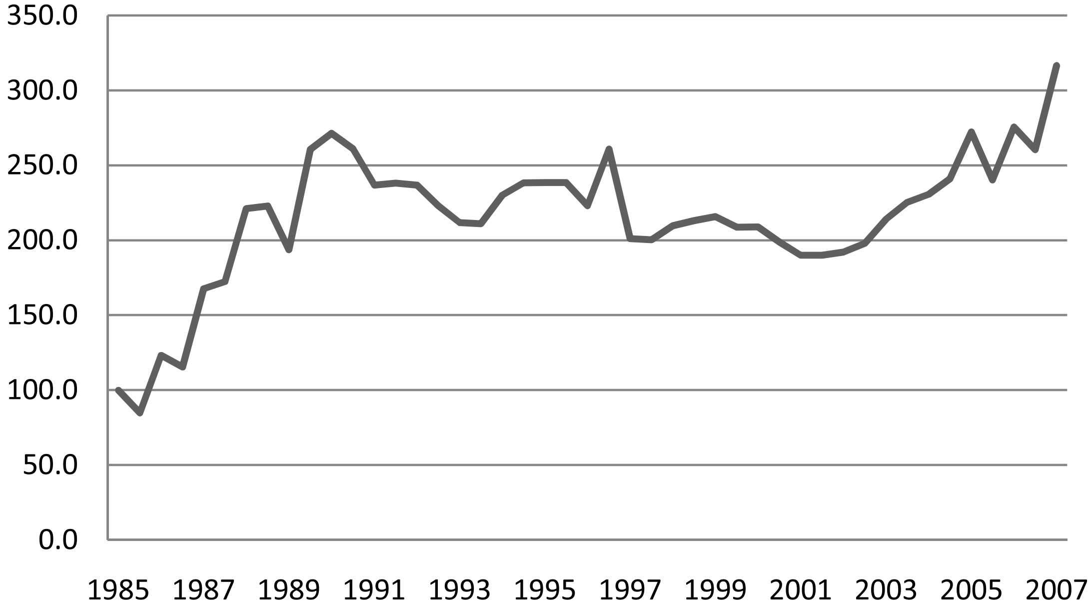


Figure 1: Semi-annual *German Art All index*. The plot shows the two-step hedonic art index based on Kräussl and van Elsland (2008). 61,135 auction records for 5,115 different German artists have been used to estimate the index between 1985 and 2007.

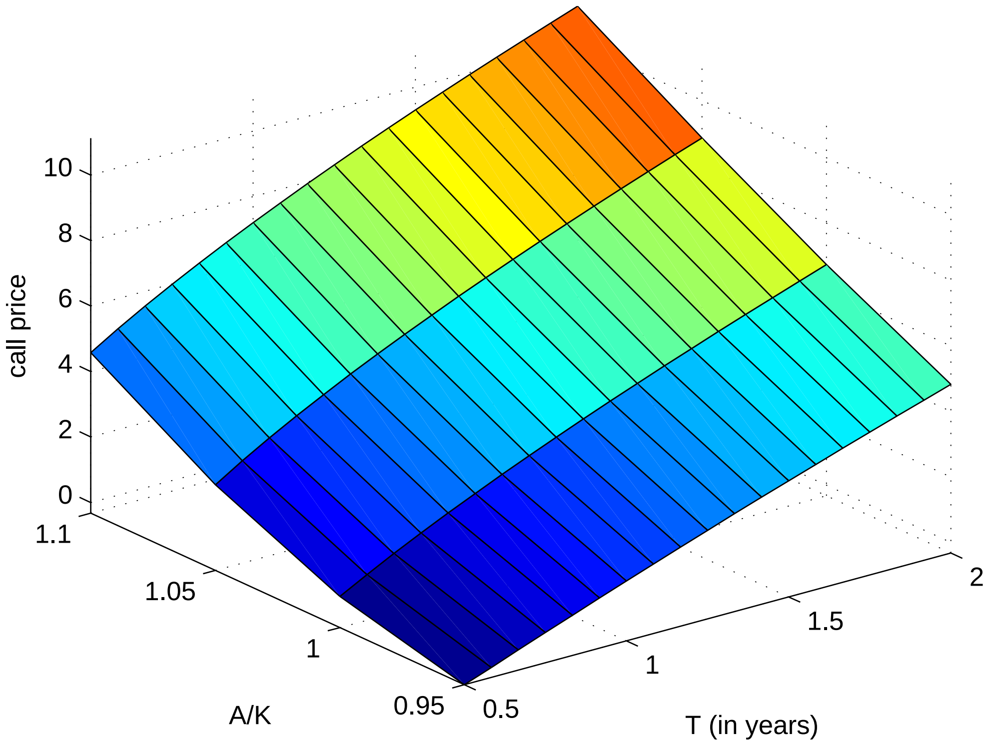


Figure 2: Equilibrium prices. The figure shows option prices in an economy with CRRA investor as function of moneyness and time to maturity. Parameter choices other than coefficient estimates (see Table 1) are $\gamma = 3$, $A_0 = 100$, and $r = 3\%$.

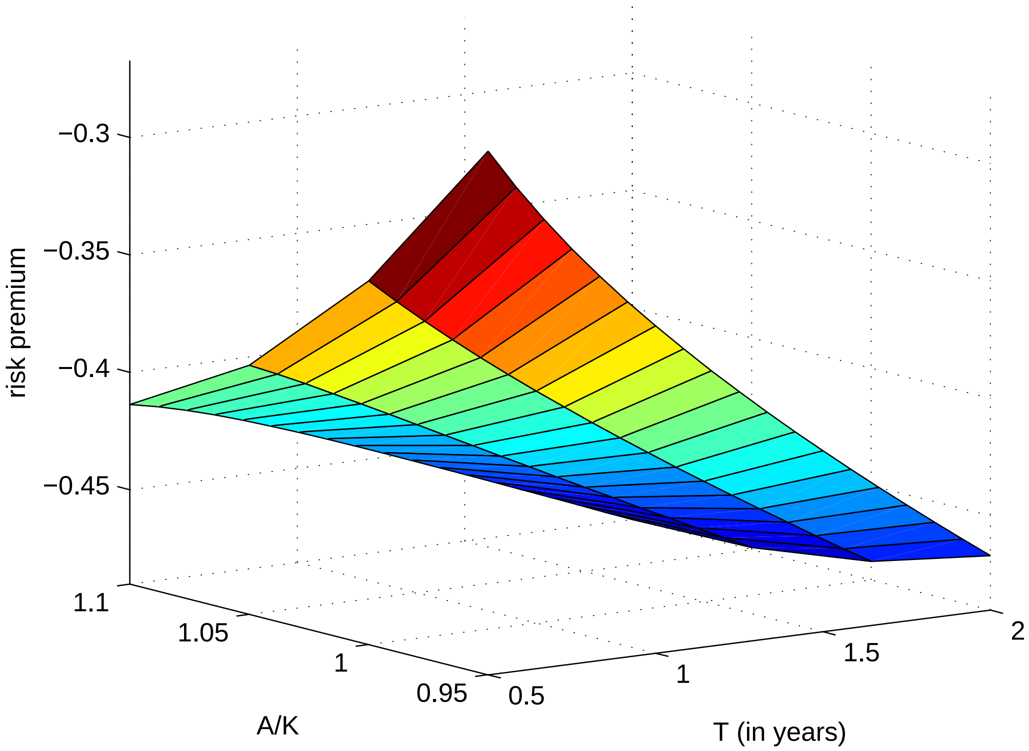
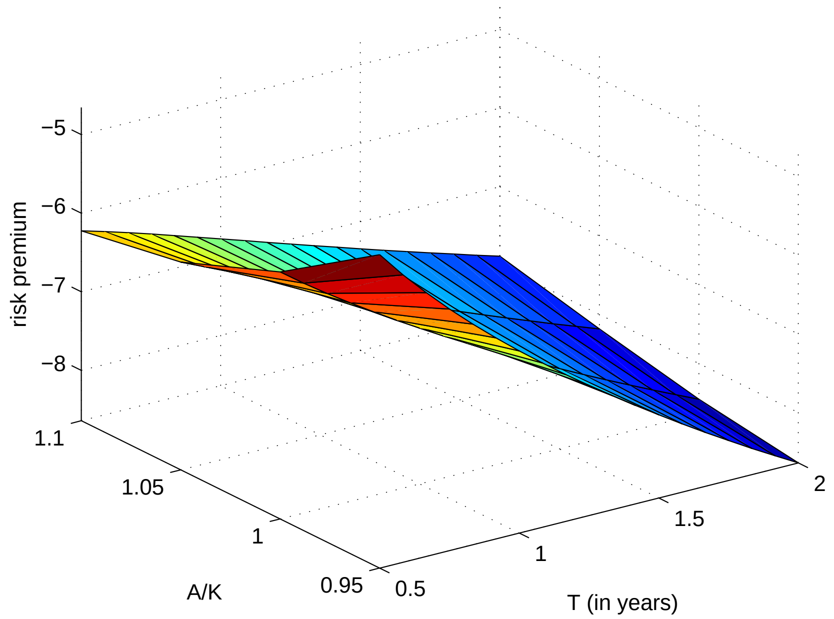
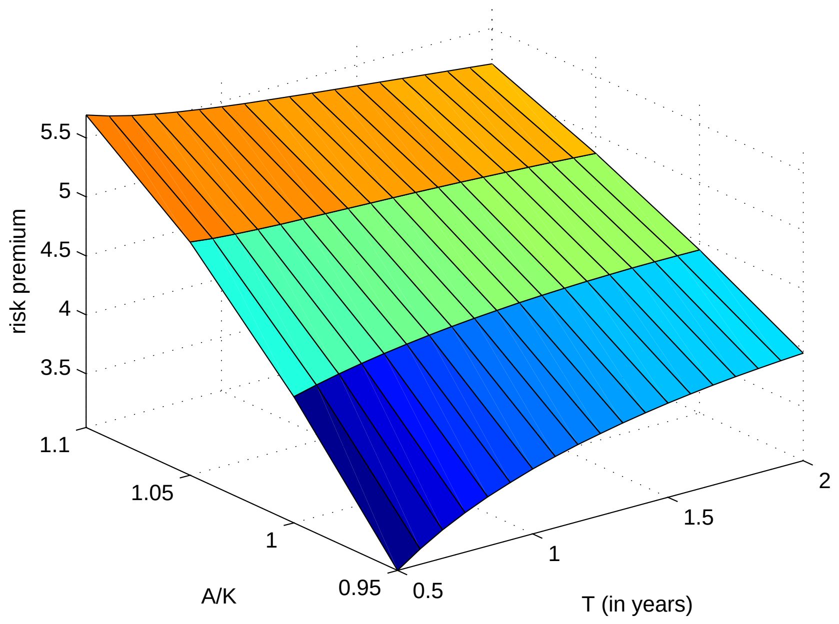


Figure 3: Risk premium. The figure shows the risk premium implied by the difference in option prices for the risk neutral investor and the CRRA agent as function of moneyness and time to maturity. Parameter choices other than coefficient estimates (see Table 1) are $\gamma = 3$, $A_0 = 100$, and $r = 3\%$.

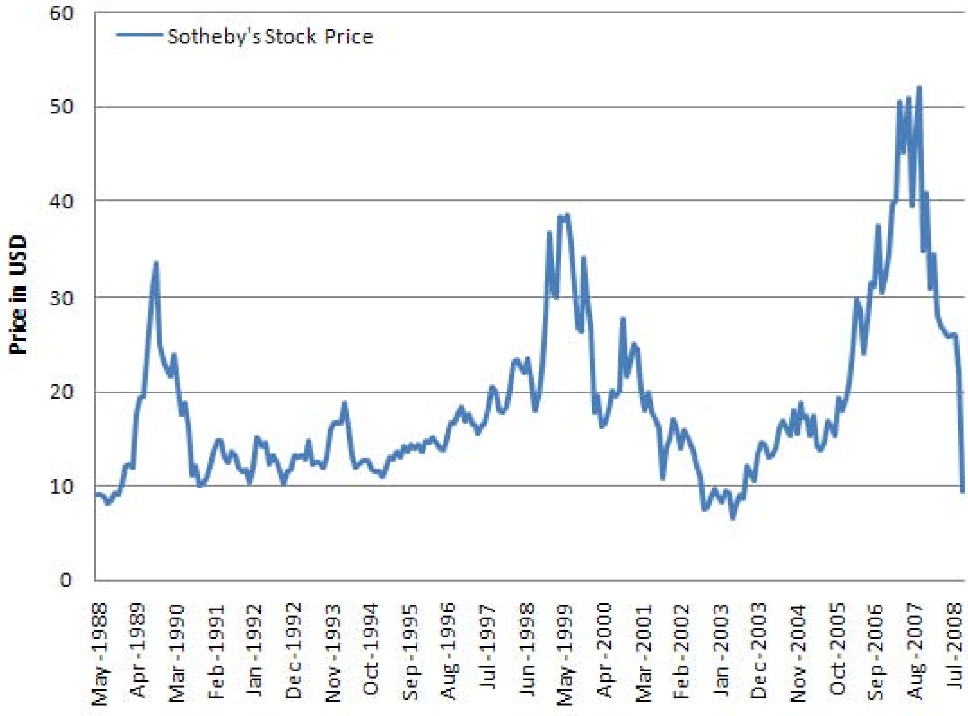


(a) $\rho = -1$

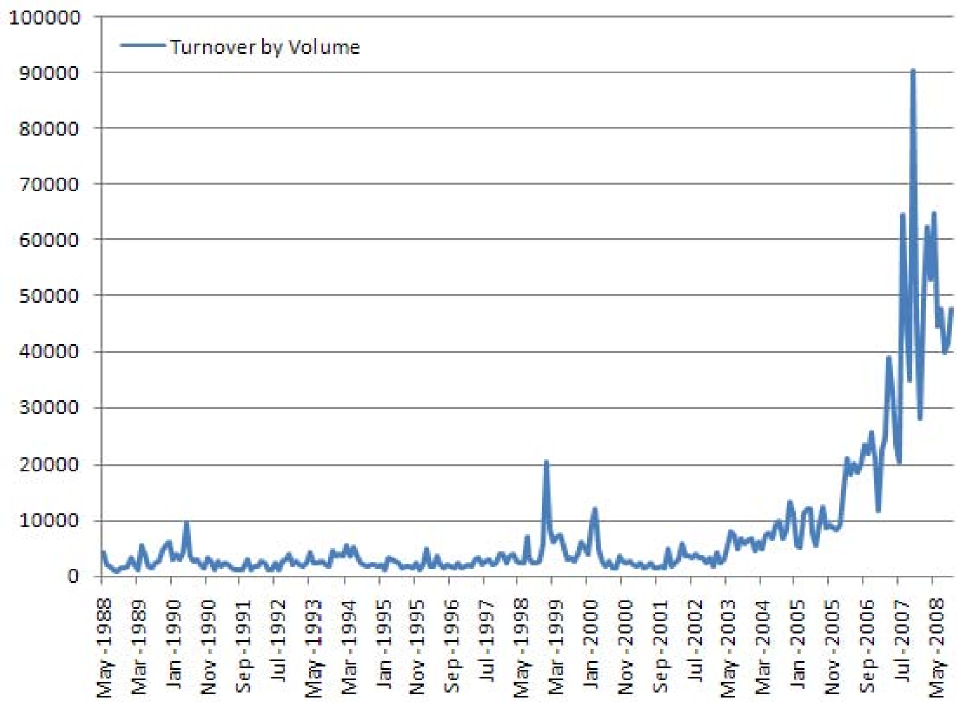


(b) $\rho = 1$

Figure 4: Risk premium and correlation. The figure shows the risk premium for different ρ implied by the difference in option prices for the risk neutral investor and the CRRA agent as function of moneyness and time to maturity. Parameter choices other than coefficient estimates (see Table 1) are $\gamma = 3$, $A_0 = 100$, and $r = 3\%$.



(a) Stock price



(b) Turnover

Figure 5: Sotheby's stock price and turnover histories.

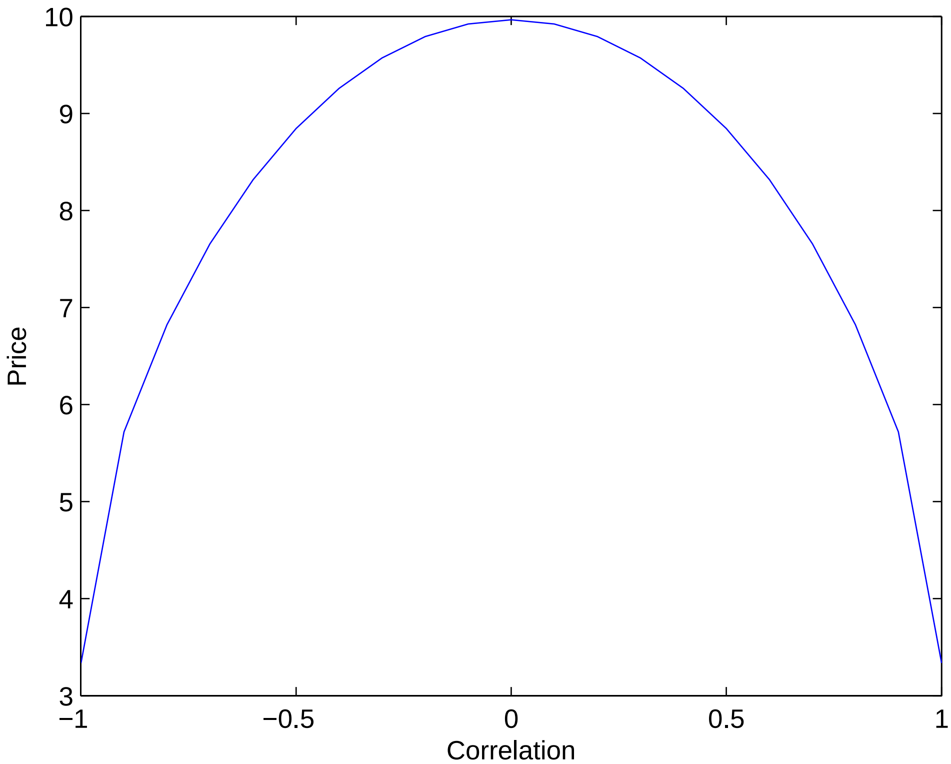


Figure 6: Correlation impact on option prices in minimal variance setup. The plot shows the effect of correlation on option prices for $\vartheta = 1$, $A/K = 1.1$ and time to maturity of one year; $r_f = 3\%$ and other parameter choices are given in Table 1.

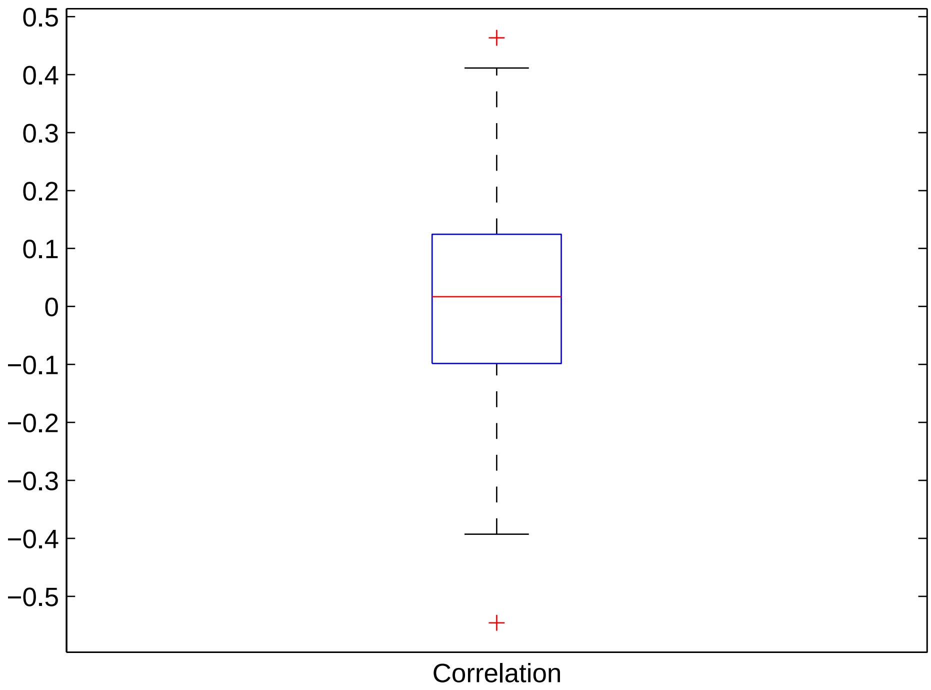


Figure 7: Distribution of correlations between art index and S&P 500 constituents. The figure shows linear correlation estimates based on semi-annual returns of 385 stocks that were part of the stock index at the end of 2007 and had been included for at least a number of years.

Table 1: Parameter estimates of art index and stock dynamics

	Art Index	S&P 500	Sotheby's Stock
Drift	0.0524	0.0888	0.0897
Diffusion	0.1787	0.1305	0.3945
Correlation		-0.0682	0.0552

Notes: Table shows the annualized drift and diffusive coefficient estimates for the semi-annual German All Art Index together with S&P 500 dynamics as well as the Sotheby's stock price. Correlations are computed with respect to the art index. Estimates are based on semi-annual observations for the time period 1988 till 2007.

Table 2: Call price as function of ϑ and moneyness

		A/K				
		0.9	0.95	1	1.05	1.1
	0	14.88	10.44	6.63	3.74	1.85
	1	24.31	19.50	14.83	10.51	6.83
ϑ	2	34.76	29.91	25.07	20.28	15.65
	3	46.20	41.35	36.49	31.65	26.81
	5	72.40	67.54	62.69	57.84	52.99
	10	162.63	157.78	152.92	148.07	143.22

Notes: Call prices as function of moneyness and risk multiple ϑ. Prices are based on the minimal variance pricing approach, where $r_f = 3\%$, $A_0 = 100$, $\rho = 5.552\%$, and $T = 1$year. Other parameter inputes are given in Table 1.

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